Radar-Diffraction Tomography Using the Modified Quasi-Linear Approximation

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Abstract—We propose a new approach for solving the scattering problem for tomographic imaging based on the modified quasi-linear approximation (MQLA). Rather than using the incident field to approximate the total field inside the scatterer as the Born approximation (BA) does, we utilize the incident field plus a secondary field that equals the incident field multiplied by a scattering coefficient, which is referred to as the quasi-linear approximation (QLA) [16], [17]. Generally, this coefficient is source dependent and hence, this approximation cannot be used to solve multiple-source problems. By invoking the localized approximation, we derive a source-independent scattering coefficient and then obtain the modified quasi-linear approximation (MQLA). Based on this approximation, we developed a new diffraction-tomography algorithm and applied it to crosshole radar tomography. This approach is rapid, since iteration techniques are not involved in the algorithm. Numerical simulations indicate that this tomography algorithm can reconstruct images with better imaging quality than the conventional BA algorithm.

Index Terms—Diffraction tomography, quasi-linear approximation, radar.

I. INTRODUCTION

GEOPHYSICAL-diffraction tomography [5], [15] (which is termed the inverse-scattering problem), as well as many other geophysical inversions [2], deals with an integration equation in the form of

\[
U(\vec{r}_s, \vec{r}_r) = U_0(\vec{r}_s, \vec{r}_r) + C \int G(\vec{r}_1 - \vec{r}_r)O(\vec{r}_1) \, d\vec{r}_1
\]

(1)

where

- \(\vec{r}_s, \vec{r}_r, \vec{r}_1\) position vectors of the source, receiver, and any point in space, respectively;
- \(C\) frequency-dependent coefficient;
- \(U\) electromagnetic, acoustic, or elastic wave field within the scatterer and it may be a vector or a scalar [2], [6];
- \(U_0\) background or incident wave field;
- \(G\) Green’s function in free space;
- \(O\) objective function.

In this paper, we focus on radar (electromagnetic wave) tomography so that all of the “\(\vec{r}\)’s” in (1) will be replaced by “\(E\)’s” that are commonly used to represent the electric fields.

However, the principle of the approximation and algorithm developed in later sections can be applied to acoustic and elastic problems as long as they are subject to (1).

Equation (1) is in a nonlinear form and cannot be solved directly. In fact, solving this equation requires knowing the total field inside the scatterer, which is impossible. Although an exact solution is unreachable, we may seek an approximate solution close to it. A common way is to linearize (1) using the Born approximation (BA) [5], [15]. However, BA is a zero-order approximation and thus is not accurate, especially when the material contrast is significant. To overcome this problem, the iterative Born inversion, the distorted BA inversion [3], [7], and the extended BA nonlinear inversion [6], [10], [11] algorithms have been developed in the past. All of these algorithms significantly enhance the image qualities compared to the conventional BA tomography. Nevertheless, all of the nonlinear or iterative methods have their own drawbacks. The most noticeable shortcoming of these methods is that they require vast computing time, which limits their practical applications. For example, the distorted BA iterative inversion requires forward modeling at each single iteration, which is very time consuming.

Can we develop an algorithm that does not need iteration but can produce better image qualities than conventional BA tomography? The quasi-linear approximation (QLA) [16], [17] seems to be a good answer to this question. Rather than using the incident field to approximate the total field within the scatterer as BA does, it uses

\[
E = (1 + \lambda)E_0
\]

in which \(\lambda\) is referred to as electrical reflectivity tensor by Zhidanov and Fang [16], [17]. In the case of scattering of scalar fields, \(\lambda\) is referred to as the scattering coefficient. However, this scattering coefficient is source dependent, which complicates our multisource inversion and has very limited applications in tomography. In this paper, we develop a source-independent scattering coefficient \(\lambda\) and thus, the modified quasi-linear approximation (MQLA). Based on this approximation, a new diffraction-tomography algorithm is formulated. Several numerical simulations are conducted to verify this algorithm and compare it with the conventional BA tomography.

II. TOMOGRAPHY ALGORITHM

A. Quasi-Linear Approximation

In order to avoid complications due to the vector nature of electromagnetic fields, a well-selected coordinate system and survey geometry are employed, which leads to a simplified
scalar-field problem [4], [13]. By doing so, the total electrical field satisfies the scalar Helmholtz equation

$$\nabla^2 E(\vec{r}) = 0$$

(2)

where

- $\vec{r}$ position vector;
- $K$ wave number in the media and is equal to $\omega \sqrt{\mu / \varepsilon}$;
- $\varepsilon$ electric permittivity;
- $\omega$ angular frequency;
- $\mu$ magnetic permeability.

Since most nonmagnetic materials have the magnetic permeability values very close to that in a vacuum, $\mu$ is assumed to be equal to $\mu_0$.

Assume that the subsurface media contain a homogeneous background medium and some heterogeneity, the scatterer that is to be imaged by diffraction tomography. By defining the objective function as $O E(\vec{r}) = 1 - K^2/K^2_0$, where $K_0$ is the wave number of the background medium, we can rewrite (2) as

$$\nabla^2 \vec{E}(\vec{r}) = K_0^2 O E(\vec{r})\vec{E}(\vec{r}).$$

(3)

It is clear that the objection function is zero in the background. The goal of diffraction tomography is to solve for the distribution of the objective function. By using the Green’s function, (3) can be reformulated in terms of the integral equation

$$E(\vec{r}) = E_0(\vec{r}) - K_0^2 \int G(|\vec{r}_1 - \vec{r}|)O(\vec{r}_1)E(\vec{r}_1) d\vec{r}_1$$

(4)

where $\vec{r}_1$ is a dummy position vector representing any point in considered space.

For most geophysical problems, multiple sources and receivers are usually utilized to enhance the data coverage. Therefore, some adjustments are made to the equation above. Consequently, we get

$$E(\vec{r}_s, \vec{r}_r) = E_0(\vec{r}_s, \vec{r}_r) - K_0^2 \int G(|\vec{r}_1 - \vec{r}_s|)O(\vec{r}_1)E(\vec{r}_1) d\vec{r}_1$$

(5)

where

- $\vec{r}_s$ and $\vec{r}_r$ source position and receiver position, respectively;
- $G(|\vec{r}_1 - \vec{r}|)$ Green’s function in free space;
- $E_0(\vec{r}_s, \vec{r}_r)$ incident field.

Equation (5), which is in the same form as (1), is a nonlinear Fredholm integral equation of the first kind. The integration is taken in the whole space. Since the objective function is zero in the background, the integration is limited within the scatterer.

An important approach to solve such nonlinear problems as (5) is to find a nearby linear problem that can be solved. The solution to this nearby linear problem is then viewed as a first approximation subject to correction of a solution to the nonlinear problem. The method commonly used for finding such a nearby linear solution is to linearize the nonlinear problem. The BA and the Rytov approximation [14] are two successful ways to solve the nonlinear problem.

First, we define the scattering field as

$$E_s(\vec{r}_s, \vec{r}_r) = E(\vec{r}_s, \vec{r}_r) - E_0(\vec{r}_s, \vec{r}_r)$$

$$= - K_0^2 \int G(|\vec{r}_1 - \vec{r}|)O(\vec{r}_1)E(\vec{r}_s, \vec{r}_1) d\vec{r}_1.$$  

(6)

Within the scatterer, if $E_s(\vec{r}_s, \vec{r}_1) \ll E_0(\vec{r}_s, \vec{r}_1)$ (that is, the scattering field is very weak), $E(\vec{r}_s, \vec{r}_1) \approx E_0(\vec{r}_s, \vec{r}_1)$. Consequently, (6) can be approximated as

$$E_s(\vec{r}_s, \vec{r}_r) = - K_0^2 \int G(|\vec{r}_1 - \vec{r}|)O(\vec{r}_1)E_0(\vec{r}_s, \vec{r}_1) d\vec{r}_1.$$  

(7)

This is the so-called BA. A similar result can be obtained in terms of the Rytov approximation [14], [15]. Although it is a very useful approach, the applicability of the assumption for the BA is very limited. In many cases, the scattering field might be strong enough to undermine its assumption. Since we know $E(\vec{r}_s, \vec{r}_1) = E_0(\vec{r}_s, \vec{r}_1) + E_s(\vec{r}_s, \vec{r}_1)$ from (6), we can find a $\lambda$ such that $E(\vec{r}_s, \vec{r}_1) \approx (1 + \lambda)E_0(\vec{r}_s, \vec{r}_1)$, which should be more accurate. This has been used in electromagnetic field modeling and inversion by Zhdanov and Fang [16], [17], where it is called quasi-linear approximation (QLA).

By definition, we obtain

$$E(\vec{r}_s, \vec{r}_1) = E_0(\vec{r}_s, \vec{r}_1) + E_s(\vec{r}_s, \vec{r}_1)$$

$$= \left[1 + \frac{E_s(\vec{r}_s, \vec{r}_1)}{E_0(\vec{r}_s, \vec{r}_1)}\right] E_0(\vec{r}_s, \vec{r}_1).$$

(8)

Ideally, $\lambda$ should be equal to $E_s(\vec{r}_s, \vec{r}_1)/E_0(\vec{r}_s, \vec{r}_1)$. Thus, (6) can be reformulated as

$$E_s(\vec{r}_s, \vec{r}_r) = - K_0^2 \int G(|\vec{r}_1 - \vec{r}|)O(\vec{r}_1)(1 + \lambda)E_0(\vec{r}_s, \vec{r}_1) d\vec{r}_1.$$  

(9)

This equation is very similar to (7) by BA linearization. If $\lambda = 0$, (9) becomes (7) and the solution of BA is the true solution. $\lambda = 0$ means $E(\vec{r}_s, \vec{r}_1) = E_0(\vec{r}_s, \vec{r}_1)$ and $E_s(\vec{r}_s, \vec{r}_1) = 0$. This situation corresponds to a wave traveling in boundless homogeneous media without any scatterer.

Let

$$O'(\vec{r}_1) = O(\vec{r}_1)(1 + \lambda).$$

(10)

Equation (9) can be simplified as

$$E_s(\vec{r}_s, \vec{r}_r) = - K_0^2 \int G(|\vec{r}_1 - \vec{r}|)O'(\vec{r}_1)E_0(\vec{r}_s, \vec{r}_1) d\vec{r}_1.$$  

(11)

Using (11) and changing $\vec{r}_r$ to $\vec{r}_2$ and $\vec{r}_1$ to another dummy vector $\vec{r}_2$, we derive

$$\lambda = \frac{E_s(\vec{r}_s, \vec{r}_1)}{E_0(\vec{r}_s, \vec{r}_1)} = \frac{- K_0^2 \int G(|\vec{r}_2 - \vec{r}_1|)O'(\vec{r}_2)E_0(\vec{r}_s, \vec{r}_2) d\vec{r}_2}{E_0(\vec{r}_s, \vec{r}_1)}.$$  

(12)
where \( \lambda \) is referred to as the scattering coefficient. Equation (11) is linear and can be solved by many inversion methods. If \( O' \) is obtained, \( \lambda \) and then the objective function \( O \) can be calculated.

Although the appearance of (11) is in the same form as (7), they are essentially different. As discussed in a later section, the \( O' \) in (11) is source dependent, while the \( O \) in (7) is not.

**B. Modified Quasi-Linear Approximation**

In practice, both \( E_0 \) and \( E_s \) are dependent on receiver position, source position, source wavelet, and frequency. Therefore, \( \lambda \) and \( O' \) should also be dependent on those parameters. For radar surveys, the sources are the same at all source points, and we may ignore the dispersion so that the effects of source wavelet and frequency can be removed away from \( \lambda \). However, it is still very complex to calculate \( \lambda \) and \( O' \) because of their dependence on source positions. Since we normally utilize multiple sources and receivers to enhance the data coverage, this method is not ready to be implemented for solving most geophysical inversion or diffraction-tomography problems. However, if we can find a source-independent approximation to \( \lambda \), it will be very useful.

It is well known that Green’s functions are singular [1] at the source point. For 3-D source representation, the singularity takes on the form

\[
\frac{1}{|r_2 - r_1|}
\]

and for two-dimensional (2-D) Green’s functions, shows the singularity

\[
\ln(K_0|r_2 - r_1|)
\]

as \( r_2 \rightarrow r_1 \). Due to the singularity, we may expect that the dominant contribution of the integration comes from the vicinity of \( r_2 = r_1 \). In fact, we can expand \( E_0(r_s, r_2) \) into a Taylor series about \( r_2 = r_1 \)

\[
E_0(r_s, r_2) = E_0(r_s, r_1) + (r_2 - r_1) \cdot \nabla E_0(r_s, r_1) + \frac{(r_2 - r_1)^2}{2} \tag{13}
\]

where \( e \) represents the error. For points within the scatterer, if the variation of \( E_0(r_s, r_2) \) is rather smooth, its gradients can be approximated to zero order. This is the localized approximation [6], [10]. Based on this assumption, we have

\[
\lambda \approx -\frac{K_0^2}{E_0(r_s, r_1)} \left[ \int G(|r_2 - r_1|)O'(r_2) \, dr_2 \right] \cdot E_0(r_s, r_1)
\]

\[
= -K_0^2 \int G(|r_2 - r_1|)O'(r_2) \, dr_2
\]

\[
= -K_0^2 \int G(|r_2 - r_1|, \omega)(1 + \lambda)O(r_2) \, dr_2
\]

\[
= -K_0^2 \int G(|r_2 - r_1|, \omega)O(r_2) \, dr_2 - K_0^2 \nabla E_0(r_s, r_1) \cdot \nabla O(r_2) \, dr_2 \tag{14}
\]

in which the Green’s function \( G \) and the objective function \( O \) are independent of the source parameters, so that the unknown \( \lambda \) should be source independent.

Since the derived \( \lambda \) is only dependent on the spatial position and irrelevant to sources, \( O' \) is also only associated with the spatial position. This makes the solutions for \( \lambda \) and \( O' \) simpler. From now on, (11) is no longer precise and should be

\[
E_s(r_s, r_1) \approx -K_0^2 \int G(|r_1 - r_1|)O'(r_1)E_0(r_s, r_1) \, dr_1. \tag{15}
\]

Consequently, we have

\[
\lambda(r') \approx -K_0^2 \int G(|r_1 - r'|)O'(r_1) \, dr_1 \tag{16}
\]

\[
O'(r_1) = O(r_1)(1 + \lambda(r_1)) \tag{17}
\]

\[
O(r_1) = \frac{O'(r_1)}{1 + \lambda(r_1)} \tag{18}
\]

and

\[
E(r_s, r_1) = E_0(r_s, r_1) + E_s(r_s, r_1) \approx [1 + \lambda(r_1)]E_0(r_s, r_1). \tag{19}
\]

Equation (19) is referred to as the modified quasi-linear approximation (MQLA). The approximation is only valid within the scatterer. However, because the integration of (9) is made within the scatterer [\( O(r_1) \) equals zero outside the scatterer], the values of \( \lambda \) outside the scatterer do not contribute to the value of the integration.

The MQLA looks similar to the extended BA [6], [10], which is based on the calculation of the internal field as the projection of the incident field onto a scattering tensor \( \hat{\Gamma}(r') \)

\[
E(r') = \hat{\Gamma}(r')E_0(r') \tag{20}
\]

where \( \hat{\Gamma} \), like the scattering coefficient \( \lambda \) in the MQLA algorithm, is the function of the material-property distribution. If we let \( 1 + \lambda(r') = \hat{\Gamma}(r') \), (19) is exactly the extended BA. Nevertheless, although both the MQLA and extended BA employ the localized approximation to do linearization, they are different. It is easy to show \( 1 + \lambda(r') \neq \hat{\Gamma}(r') \). The extended BA is derived by successive iteration, while MQLA is achieved without iteration.

**C. Implementation**

Our tomography implementation, similar to the QLA inversion [17], is divided into the following three steps:

1) calculating \( O'(r_1) \) by (15);
2) computing \( \lambda(r_1) \) by (16);
3) inverting \( O(r_1) \) (18).

When calculating the scattering coefficient, we need to calculate the Green’s function. In 3-D free space, the Green’s function is

\[
G(|r_1 - r'|) = \frac{e^{iK_0|r_1 - r'|}}{4\pi|r_1 - r'|}
\]
and in 2-D space

\[ G(p_1 - \bar{p}) = \frac{i}{4} H_0^{(1)}(K_0|p_1 - \bar{p}|) \]

\[ = \frac{1}{4\pi^2} \int \frac{e^{i\bar{K}\cdot(r - p)}}{K_0^2 - K^2} d\bar{K} \]

where \( H_0^{(1)} \) is the zero-order Hankel function of first kind and \( \bar{K} = (K_x, K_z) \). The calculation of a 2-D Green’s function is much more complicated than that of a 3-D Green’s function.

For a 2-D crosshole configuration, \( \bar{p} = (x, z) \), and we can substitute \( \bar{p}_s \) and \( \bar{p}_r \) with \( (x_s, z_s) \) and \( (x_r, z_r) \), because the horizontal coordinates of the source and receiver are assumed to be constants. The form of (11) is the same as the equation for the BA method. Hence, we may use the conventional BA tomography technique for inverting \( O' \). The conventional backpropagation formula for a 2-D crosshole configuration [8] is

\[ O'(x, z) = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{[k_s \gamma_s + k_r \gamma_r]}{K_0^2} E_s(k_s, k_r) \]

\[ \cdot e^{-i(k_s \gamma_s d_s + k_r \gamma_r d_r)} e^{i[(\gamma_s \gamma_r)^x + (k_s + k_r)^z]} dk_s dk_r \]

(21)

where

- \( k_s \) and \( k_r \) wave numbers corresponding to source depth \( z_s \) and receiver depth \( z_r \);
- \( d_s \) and \( d_r \) distances between the origin and the source well and receiver well, respectively;

and

\[ E_s(k_s, k_r) = \text{FT}[E_s(z_s, z_r)] \]

\[ \gamma_s = \sqrt{K_0^2 - k_s^2} \]

\[ \gamma_r = \sqrt{K_0^2 - k_r^2} \]

where FT denotes Fourier transform.

For each frequency, a tomographic image of the media can be produced. Averaging the images of a frequency band will improve the image quality and suppress the noise [12].

The main difference between the MQLA algorithm and the QLA algorithm is that the former is source independent and therefore can be used to solve multisource radar problems, whereas the latter is source dependent and not suitable for multisource problems. In addition, the QLA was developed for general vector EM problems, while the MQLA for specific radar problems in which high-frequency (commonly 5–1000 MHz) scalar wave fields and good dielectric media are assumed.

III. NUMERICAL SIMULATIONS AND ANALYSIS

A. Numerical Verification and Testing of Adaptability to Scattering

Our first experiment is aimed at verifying the MQLA algorithm, testing its adaptability to scattering, and comparing it with the conventional BA algorithm. The 2-D synthetic model shown in Fig. 1 is a 50.0 m x 50.0 m area that contains a 2.0 m x 4.0 m rectangular scatterer. The “m” used in this paper represents meters. We take the upper left corner of this model as the origin of the Cartesian coordinate system, in which the \( z \) axis points downward, and the \( x \) axis points to the right. This model is a generic model because we use it to conduct three tests, in which we change only the material property of the scatterer, while the sizes and the background remain unchanged. The relative dielectric constant is 9.00 in the background, and we assume perfect dielectric media (the conductivity \( \sigma = 0.00 \)). Let the relative dielectric constant of the scatterer \( \epsilon_r \) be 7.44, 6.25, and 4.00, respectively, for the three tests. These dielectric constants create 10%, 20%, and 50% velocity contrasts. According to the definition, the correspondent objective functions are 0.174, 0.305, and 0.556 for the scatterer. The simulations take crosshole geometry. The transmitters are placed in the left well at a 1.0-m interval, so that there are 51 transmitter positions. The same amount of receivers are put in the opposite well at the same interval. The main frequency of the source used is 75 MHz (megahertz). All of the synthetic data in this paper are generated by finite-difference modeling SUFDMOD, which was developed by the Center for Wave Phenomenon, Colorado School of Mines, Golden, to simulate acoustic propagation. Because lossless media are assumed, the scalar EM equation is in the same form as the acoustic wave equation. The frequency band used for tomography is 60–90 MHz, with a sampling interval of 1.64 MHz.

All imaging results produced by both the MQLA and BA algorithms are shown in Figs. 2, 3, 5, 6, 8, and 9. The cross sections, at horizontal distances of 25.0 m, are displayed in Figs. 4, 7, and 10. It is noticed that there are diagonal extensions at the four corners of the scatterer, and the scatterer is widened in the horizontal direction in the images. These effects are caused by...
Fig. 2. Tomographic image of test 1 using the BA algorithm. In the associated synthetic model, the dielectric constant for the scatterer $\varepsilon_r = 7.44$ and the corresponding objective function is 0.174.

Fig. 3. Tomographic image of test 1 using the MQLA algorithm. In the associated synthetic model, the dielectric constant for the scatterer $\varepsilon_r = 7.44$ and the corresponding objective function is 0.174.
Fig. 4. Vertical section comparison for test 1. The horizontal distances of all vertical sections are at 25.0 m.

Fig. 5. Tomographic image of test 2 using the BA algorithm. In the associated synthetic model, the dielectric constant for the scatterer $\varepsilon_r = 6.25$ and the corresponding objective function is 0.305.
Fig. 6. Tomographic image of test 2 using the MQLA algorithm. In the associated synthetic model, the dielectric constant for the scatterer $\epsilon_r = 6.25$ and the corresponding objective function is 0.305.

Fig. 7. Vertical section comparison for test 2. The horizontal distances of all vertical sections are at 25.0 m.
Fig. 8. Tomographic image of test 3 using the BA algorithm. In the associated synthetic model, the dielectric constant for the scatterer $\varepsilon_r = 4.00$ and the corresponding objective function is 0.556.

Fig. 9. Tomographic image of test 3 using the MQLA algorithm. In the associated synthetic model, the dielectric constant for the scatterer $\varepsilon_r = 4.00$ and the corresponding objective function is 0.556.
the limited coverage. On the one hand, we only utilize a certain frequency band so that we lose the information at higher frequencies. On the other hand, the crosshole geometry has limited coverage in the horizontal wave-number direction [5], which means the horizontal resolution is worse than the vertical resolution in crosshole tomography.

These results indicate that, when the velocity contrasts are 10% and 20%, both the BA and MQLA methods can define the shape of the scatterer. However, the inverted-objective function values by the MQLA method are much closer to the true model values than those by the BA method. This is very clear in Figs. 4 and 7. When the velocity contrast is as high as 50%, the scatterer reconstructed by the BA method in Fig. 8 is a low-velocity scatterer (the objective-function values are less than zero), which is contradictory to the true model. There is a high-velocity scatterer by the MQLA method in Fig. 9, but it is narrower than the true scatterer.

The results of this experiment are consistent with the theory. The result of the BA algorithm is equivalent to $O(f_r)$, which is the semifinished product of the MQLA algorithm. To get the final image, further refining is conducted (calculating the scattering coefficient and removing its effect from the final product (18)). This is very important for diffraction tomography, because the scattering exists as long as heterogeneity is present.

Although the MQLA and BA algorithms are independent from each other, we obtain a sense that the MQLA algorithm performs a scattering correction after the BA tomography. The BA algorithm assumes no scattering inside the scatterer, which renders it inaccurate. After the scattering correction (although only an approximate correction) the total field inside the scatterer should be closer to the true field and the resulted image should be closer to the true geological model.

However, when the material contrast reaches a certain level, the assumption for the MQLA algorithm is also undermined.
Fig. 12. Tomographic image of three-scatterer model using the BA algorithm.

Fig. 13. Tomographic image of three-scatterer model using the MQLA algorithm.
Fig. 14. Cross section comparison of the objective function images at a horizontal distance of 21.0 m. This section cuts through the left two scatterers.

Fig. 15. Cross section comparison of the objective-function images at a horizontal distance of 29.0 m. This section cuts through the right scatterer.
That is, the localized approximation no longer holds. This is why the image of MQLA in Fig. 9 is poorly resolved but is still better than the BA result in Fig. 8.

B. Multiple Scatters

The second experiment is to test the MQLA algorithm’s applicability to multiple-scatterer problems. The model is still a 50.0 m × 50.0 m area that contains three rectangular scatterers. The two scatterers on the left side are 2.0 m in height and 4.0 m in width, and the other one is 4.0 m in height and 2.0 m in width, as displayed in Fig. 11. The coordinate system, source, and survey geometry are the same as those in the first experiment. The relative dielectric constant is 9.00 for the background and 7.44 within the scatterers. The tomographic results produced by the BA and MQLA algorithms are shown in Figs. 12 and 13, respectively. As discussed in the first experiment, the diagonal extensions at the corners, and the widening effect in the horizontal direction of the scatterers in both images, are due to the limited wave-number coverage.

In Fig. 12, the two scatterers on the left side are clearly resolved, but the one on the right side is “dim” and poorly defined. The reason is that the waves propagate from the left side and the right scatterer is in the shadows of the two scatterers on the left side. This is expected for any scatter lying in a shadow. This also occurs in Fig. 13, although the three scatterers in this image are much clearer compared to those in Fig. 12. The most obvious improvement is that the right scatterer is relatively clearer, although significantly widened in the horizontal direction. The vertical sections at the horizontal distances of 21.0 and 29.0 m are displayed in Figs. 14 and 15, where it can be seen that the objective function values in the MQLA algorithm are much closer to the true model than those in BA algorithm. The scatterer on the right side in Fig. 13 is not as “bright” as the other two, which indicates that the “shading” effect still exists. This experiment shows that when there is more than one scatterer in the media, the MQLA algorithm is better than the conventional BA tomography, and scatterers illuminated directly by sources are better resolved than those shaded by others.

IV. CONCLUSIONS

A new approximation is developed and applied in radar-diffraction tomography. The numerical simulations indicate that the algorithm based on this approximation is much more accurate than conventional BA, while it needs only modestly more computing time. Synthetic simulations show that when the material contrast is below 20%, the MLA algorithm can resolve the scatterer well, and when more than one scatterer exists, those not illuminated by sources directly are less resolved (dimmer) than those directly exposed to the sources.

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