The effect of changing scatterer positions on acoustic time-reversal refocusing in a 2D urban environment at low frequencies

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Abstract
In a previous work, we considered time-reversal refocusing for localizing a sound source in a highly reverberant urban environment with non-line-of-sight (NLOS) receivers. The approach involved a virtual (computer-based) time-reversal propagation calculation using \textit{a priori} knowledge of the receiver and scatterer positions. Among the errors that affect the quality of refocusing is the imperfect accuracy of the assumed scatterer locations. Some practical analogues of changing scatterer positions are (a) the building coordinate errors (or mistakes) and (b) the random movement of vehicles on the street of an urban setting. We address these issues by conducting a time-reversal analysis of a set of numerically generated synthetic acoustic pressure recordings after intentionally introducing errors by altering the position of the scatterers. The numerical experiment results indicate that, for a realistically complicated urban setting, coordinate errors in the position of only a few scatterers have little detrimental effect. The effect of coordinate errors in one or a few scatterers is suppressed by the constraints imposed by correct propagation features (both kinematic and dynamic) generated by multiple reflections and diffractions from other precisely located scatterers. The dependence of source localization accuracy on the relative error in scatterer coordinates, the relative number of scatterers with coordinate errors and the number of receivers is quantitatively assessed in this paper. In general, time-reversal refocusing with a large number of receivers can compensate for more severe errors. Even for only three non-line-of-sight receivers, building location errors in the order of 3 m still give good results. Adding small scatterers (like vehicles) close to a receiver is more critical than adding scatterers relatively far from the receivers.

Keywords: acoustic wave propagation, time reversal, scattering, source detection, numerical modelling, urban environment

Introduction
The presence of buildings greatly complicates sound propagation in an urban environment (e.g., Oldham and Radwan 1994, Picaut et al 1999, Van Renterghem et al 2006). Sound waves reflect from building walls and diffract around corners. Because of the many echoes and diffractions, it is difficult for a listener to locate the source of the sound waves, especially if the source is not directly visible (non-line-of-sight, or NLOS). Locating a sound source in a noisy urban setting is of great interest in many civil and military engineering applications. Previously, time-reversal processing has been successfully applied to source localization and medium imaging in...
many research and engineering disciplines such as biomedical imaging (Thomas et al 1996, Xu and Wang 2004) and underwater ocean acoustic communication (Fink 1997, Kuperman et al 1998), where it was necessary to overcome complex propagation effects. Earlier papers by the present authors have suggested using time-reversal processing to locate acoustic sources in an urban environment (Liu and Albert 2004, Albert et al 2005). There are several potential errors that can affect the quality of time-reversal refocusing in an urban environment. In this paper, we focus on the following: (1) measurement errors and mistakes in building coordinates, (2) random addition or elimination of small scatterers in a given urban setting when vehicles constantly and randomly move in and out of street blocks.

Another major concern with time reversal for this kind of application is its robustness when the propagation medium is moving or varies between the times of forward and reverse back propagation. These phenomena may occur as a result of wind channelling and turbulence in street canyons (USEPA 1998, Van Renterghem et al 2006), and have the effect of destroying spatial and temporal reciprocities necessary for time-reversal processing (Roux et al 2004). Absorption of sound by air and surfaces also destroys spatial reciprocity. Although these limitations to reciprocity are potentially important, we place emphasis here on the sources of error mentioned in the preceding paragraph. This may be justified by the low frequency range involved, for which one would expect absorption and scattering by turbulence to be less significant.

Our methodology in this paper is based on numerical simulation. We concentrate on the investigation of acoustic wave forward propagation and time-reversed refocusing in a simplified two-dimensional (2D) model of a typical urban environment. The approach and conclusions can be readily extended to three-dimensional (3D) cases to study more complicated and realistic cases.

Forward modelling

The finite-difference, time-domain (FDTD) method is based on the expression of acoustic propagation as a set of first-order, velocity–pressure-coupled differential equations (Wang 1996, Ostashev et al 2005). To approximate the derivatives in the acoustic wave equation with finite differences, a staggered difference algorithm (Yee 1966) is used in a two-dimensional spatial domain (Liu and Albert 2006). The marching is also staggered in the time domain between the computations of the pressure and the particle velocity. The stretched-coordinate version (Chew and Weedon 1994) of the original perfectly matched layer technique (Berenger 1994) was adapted for the absorption boundary condition and achieved highly effective suppression of unwanted reflections from the domain boundaries.

To keep computational demands reasonable, the real three-dimensional world is simplified to a two-dimensional model. Buildings are treated as solid blocks in the calculations to speed up the geometric input to the model. Because of the 2D approximation, acoustic phenomena involving vertical propagation of energy, i.e. diffraction over the top of the buildings, are not included in the calculations. However, since the emphasis of this paper is on investigating the robustness of time reversal to see its degree of tolerance to scatterer positioning errors, it is not our intention to calculate all the details of sound propagation features in urban environments as long as the same modelling algorithm is used for both forward and time-reversed computations. The details of the 2D model algorithm and the justification of approximations have been discussed in a previous study (Liu and Albert 2006).

Discrete urban model

Our numerical experiments were based on the building locations in a full-scale, artificial training village. We have previously recorded impulsive signals in this environment. The geometry and location of the buildings, as well as source and receiver network locations used in the calculations, are shown in figure 1 and discussed in a previous study (Albert et al 2005). This flat area has 15 closely spaced concrete buildings (A–O) arranged in a 200 × 140 m² area. The 2D domain for the FDTD calculations consisted of a 667 × 467 grid. Each grid cell was 0.3 m × 0.3 m. A total of 2000 time steps, each lasting 0.2 ms, were performed to span an elapsed time of 0.4 s. The impulsive sound source is at the grid location of (213, 330) (the asterisk in figure 1) with a central frequency of 150 Hz. For these low frequencies, atmosphere refraction and absorption are less important.

In this numerical study on the robustness of time-reversal refocusing, we used a source known as the Ricker wavelet (Sheriff and Geldart 1995) with central frequency 150 Hz. The forward-propagating waves can be temporally recorded at any location in the model domain, including 14 locations where microphone sensors were placed in the real-world

![Figure 1. Locations of the buildings, acoustic source and receivers for the numerical experiments. The solid dots are the locations of microphones used in an actual field experiment, whereas the white circles are additional virtual receiving points used only in the numerical study.](image-url)
measurements and 91 other virtual locations for later use in the time-reversal process. These 91 virtual receiving points surround all four sides of the village-building clutter, and also include three additional linear arrays in the EW direction (horizontal direction in figure 1) and two linear arrays in the NS (vertical) direction. The relative positions of buildings, source and all these 105 receiving points are shown in figure 1.

Forward acoustic wave propagation in the urban model

Figure 2 shows acoustic wavefield snapshots of the forward-propagation simulation at elapsed times of 10, 50, 100, 150, 200, 250, 300, 350 and 400 ms. The snapshots clearly show strong scattering from the buildings. Diffraction produces secondary sources, and long-wavelength reverberation occurs in the street canyons. The wavefield from this forward modelling exercise, as recorded at the 105 receiving points, will be used in the following section to study the time-reversal refocusing process.

Figure 3 shows one example of the time series calculated from two linear arrays of receivers: the central NS array located just east of building F running south from just south of building B through the main EW street towards building L, and the southernmost EW array running west to east and stopping near building I (figure 1).

The time records from the NS array (figure 3(a)) clearly show that the first three channels (0–2) are line-of-sight (LOS) locations with respect to the source. They have the strongest first direct arrivals and the shortest travel time. For these three channels, a more careful examination can lead to the recognition of street reverberation with a much longer period (about 60 ms). The next four channels (3–6) have much smaller amplitudes, due mainly to the blockage of direct wave energy by building F. The last four channels (7–10) see a moderate recovery of wave energy, due to some sound waves passing through the gap between buildings B and F and diffracting to the south of building F.

For the time records (figure 3(b)) of the EW array south of buildings J, K and H, the amplitudes of all channels are much smaller, with channels 0–1 and 4–5 having the shortest travel time because their locations have the shortest propagation distances from the source. Channels 2 and 3 have smaller amplitude and longer travel time since the sound waves have a longer propagation path to reach these two locations.
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**Figure 3.** Synthetic time records recorded at receiver arrays by the central NS array just east of building F starting from the main street down south (a) and the southernmost EW array from west to east (b).

**Figure 4.** Snapshots of acoustic signal power for time-reversal refocusing at 10, 200, 300 and 400 ms. All 105 receivers shown in figure 1 were used in this calculation.

**Time-reversal refocusing with scatterer position changes**

After computing the simulated acoustic wavefield at all 105 receiving points, we conducted the time-reversal back propagation processing in the same model domain. Without changing any building positions or adding any scatterers, we reversed all 105 time sequence records to feed into the model, thereby generating the back propagated wavefield snapshots as shown in figure 4.
The acoustic energy refocused at the original source
location (213, 330) with excellent resolution. The amplitude
of the time-reversal refocused wave can reach 35.4% (a peak
value of $5.147 \times 10^5$ Pa of the prescribed original source
amplitude of $1.4545 \times 10^6$ Pa). The spatial width of the peak
(the horizontal distance for the amplitude dropping from the
maximum to 50% of the maximum) is only about two grids
(0.6 m). We have taken this result as the baseline for
a comparison with time-reversal refocusing under other
scenarios in which a variety of building position changes are
made.

**Time-reversal processing with all 105 receivers and building
repositioning and removal**

Since the source was located in the northwest portion of the
domain between buildings A and E on the main street, and
the acoustic wave mainly propagates to the centre and rest
of the domain to the southeast, we decided to focus on studying
the effects of buildings F and B which are adjacent to buildings
A and E, respectively, and potentially have the most significant
impact on the propagation and diffraction of the wavefield.
Using recordings from all 105 receivers to carry out the time
reversal, we have investigated the following virtual scenarios.

1. Building F moved north into the EW main street by 5 m.
2. Building F was completely removed from the model.
3. Buildings F and B were completely removed from the
model.

Figure 5 compares these four scenarios of building
repositioning and removal with the baseline scenario. The
refocusing accuracy is essentially minimal. The amplitude
of the refocused sound level drops from $5.147 \times 10^5$ Pa
to $4.495 \times 10^5$ Pa of the worst scenario (buildings F and
B being completely removed), only a 12% relative change,
whereas the spatial width has no noticeable changes at all.
The comparison presented in figure 5 demonstrates that
if a large number of receivers in the sensor network are
available, wavefield coverage with this dense receiver network
suppresses significant errors in scatterer positions. Only one
or a couple of changes in building positions has a little effect
on the robustness of time-reversal refocusing.

**Time-reversal processing with three NLOS receivers and
building repositioning**

Having demonstrated that a large number of receivers can
substantially suppress the position error in time reversal, we
want to see how severe the scatter position error is when only
a few sensors are used. This test uses recordings from only
three NLOS receivers (08, 16 and 20, see figure 1) to carry out
time-reversal back propagation.

Figures 6 and 7 show time-reversal refocusing at the final
stage based on recordings from these three receivers. Six
scenarios were generated for the position changes of building
F; the scenario for no position change for all buildings serves
as the baseline for the time-reversal process by using only
records from these three receivers (scenario 1). The peak
value of the time-reversed refocusing is $1.10 \times 10^4$ Pa and
the spatial width is about 2.4 m (eight grids). Then, building
F is moved north for 1, 2, 3, 4 and 5 m (scenarios 2–6).
Figure 6(a) shows a snapshot of the absolute pressure at the
final step of the time reversal without any building adjustments
(scenario 1). The peak acoustic pressures have always refocused at the original source location ($N_x = 213$). The peak value drops from 10.6 kPa for the scenario of no movement to 9.8, 8.0, 6.7, 6.1 and 5.7 kPa, and the
Figure 6. (a) Time-reversal refocusing based on recordings at 08, 16 and 20 for the scenario of no position changes of the building. (b) Refocusing along an EW profile through the source location ($Ny = 330$) for the baseline (no movement) scenario and for the position of building F moving north by 1, 2, 3, 4 and 5 m.

Figure 7. Time-reversal refocusing at the final calculation step based on records at receiver points 08, 16 and 20. The vertical axis represents received signal energy. The baseline scenario of no change in the building positions is compared to building F moving north by 1, 2, 3, 4 and 5 m. Time-reversal refocusing breaks down when building F was moved 3 m or further to the north.

Spatial width degraded from 2.4 to 2.4, 2.7, 3.0, 4.8 and 3.6 m for the scenarios of building F moved north for 1, 2, 3, 4 and 5 m, respectively. The maximum drop is 46% in amplitude, and the spatial width is doubled. The variations of both parameters indicate rapid degradation of the refocusing sharpness. Also, as another indicator of robustness, along the EW profile going through the source location the ratio of the (side lobe)/(peak) can reach about 50% for the scenarios in which building F moved more than 4 m (figure 6(b)). More importantly, from the 3D visualizations of the distribution of...
Figure 8. (a) Time-reversal refocusing based on recordings at nine receiver positions (including receivers 08, 16 and 20) for the scenario of building F moving north by 4 m. (b) A comparison of building coordinate error effects on time-reversal refocusing along an EW profile through the source location ($N_y = 330$) for the position of building F moving north by 4 m.

Figure 9. A comparison of building coordinate error effects on time-reversal refocusing along an EW profile through the source location ($N_y = 330$) for the position of building A moving west by 1, 3 and 5 m and building E moving south by 1, 3 and 5 m.

The effect of number of receivers used in time-reversal refocusing and repositioning of other buildings

As demonstrated above, using more receivers in the time-reversal refocusing should have the advantage of a better refocusing. The degree of refocusing for any number of receivers between 105 and 3 should have an intermediate degree of refocusing, just by an intuitive guess, and proved by the following test. Figure 8(a) shows the scenario of time-reversal refocusing with nine receivers, and with building F moving north by 4 m. Figure 8(b) compared the refocusing along the EW profile going through the source location by using nine and three receivers (3 used in figure 6). It is clear that refocusing by using nine receivers is noticeably better than that using only three receivers. For the scenario of building F moving north by 4 m, the ratio of the peak at the source location and the highest side lobe increased from 2.3 by using three receivers to 3.2 by using nine receivers, there is a 1.4 times increase in the accuracy of refocusing.

A further test on building coordinate errors on time-reversal refocusing was carried out by moving the two buildings close to the source, buildings A and E separately. Building A was moved west by 1, 3 and 5 m, and building E south by 1, 3 and 5 m. Figure 9 shows the refocusing results along the same EW profile crossing the source. It appears that there are very little effects on time-reversal refocusing by

the absolute sound level over the model domain (figure 7), we can see that when the building is moved less than 3 m (about 25% of the side length of building F), the time-reversed back propagation can always refocus the sound energy to the original source location. When the shift is 4 m or greater, the peak at the original source location is not the global maximum anymore.

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Moving building A or E up to 5 m. However, it also shows that moving building E might have a more significant effect than moving building A, by seeing that the amplitudes of the main peak values for moving building E are lower than those for moving building A, and the rate of the peak value dropping due to larger errors is more noticeable.

**Time-reversal processing with three NLOS receivers and the addition of scatterers**

To look into the problem of the effect of vehicles on time-reversal refocusing in an urban environment, we have also simulated the following four scenarios. First, we have added six obstacles with a size similar to automotive vehicles on the main street; their locations were randomly selected (figure 10). These obstacles were placed in the model domain relatively far from the three receivers (08, 16 and 20) used in time reversal. For the following three scenarios, two additional obstacles were added close to one of the three receivers to study the effect of having vehicles close to the receiver. It is noteworthy that these obstacles are 2D columns in the model; actual cars with limited height should have even less effect so that the modelling result here may be regarded as a conservative estimate of the refocusing degradation.

**Figure 10.** A comparison of the sound level results for the final step of time-reversal back propagation with four scenarios: (a) six obstacles added between the source and receivers but far from all receivers, (b) six obstacles plus an additional two obstacles close to 08, (c) six obstacles plus an additional two obstacles close to 16 and (d) six obstacles plus an additional two obstacles close to 20.

Figure 10 shows the spatial distributions in the final stage of the time-reversal refocusing for these four scenarios. A comparison of figures 10(a) and 6(a) shows no significant difference in sound level refocusing after adding the six car-like obstacles. A more quantitative examination of the sound level at the original location hardly reveals any noticeable changes between the scenarios with and without the six obstacles in the streets, thereby demonstrating that adding small obstacles far from the receivers has little effect on the robustness of time reversal. However, when two more obstacles are added close to one of the receivers, significant changes in sound level distribution are observed. Scenario (b) (six obstacles plus an additional two more obstacles close to 08) has the most obvious effect on refocusing: the sound level is dropped 49% at the original source location (figure 10(b)). For scenario (c) (six obstacles plus an additional two more obstacles close to receiver 16) and scenario (d) (six obstacles plus an additional 2 more obstacles close to receiver 20), the pressure level drops 29% and 2%, respectively (figures 10(c) and (d)). The reason for this large difference in the influence of the obstacles seems to be associated with the fact that
receiver 08 is the closest scenario to the line of sight to the source, whereas receiver 20 is the farthest scenario from the line of sight to the source. Receiver 08’s contribution to the refocusing is much larger than receiver 20’s, and receiver 16 is intermediate for both closeness to the line of sight and contribution to the time-reversal refocusing.

Conclusion

A previous study (Albert et al 2005) demonstrated that time-reversal processing can recover an acoustic source location in an urban area; nevertheless, the robustness of this technique with regard to errors in building locations and the presence of additional obstacles was still unknown. In this paper, we have presented a series of numerical experiments that demonstrate the following.

1. Time-reversal refocusing is quite tolerant of building location errors if a large number of receivers are employed, as with the 105 receivers considered here.
2. A larger number of receivers in the network will substantially improve the source localization accuracy in time reversal, as indicated by the comparison of the scenarios using 105 receivers versus only three NLOS receivers.
3. Even with only a few NLOS receivers, time-reversal refocusing is still possible, though with degraded resolution, if the building position errors are minor.
4. Addition of obstacles in locations far from the receivers that participate in the time-reversal refocusing has an insignificant effect on the accuracy of source localization. In contrast, obstacle additions close to the participating receivers cause substantial degradations.

The results of this study are very encouraging for using time reversal for source localization in a relatively dynamic environment. In real-world situations, the accuracy should be even better than the 2D model results since all buildings have a finite height and a portion of sound wave energy can pass over the building roofs. In addition, more secondary diffraction sources from more building corners should further increase the effective aperture of the sensor network. To simulate all details of the propagation, time-reversal techniques should be studied in a full three-dimensional manner as was done for electromagnetic waves (e.g., Liu and Arcone 2005). However, the computational cost of 3D back propagation in the model domain is much higher than the 2D case. Statistical analysis (Alfaro Vigo et al 2004) and a broader frequency bandwidth should also be considered in future studies. However, the degree of applicability of the time-reversal method at higher frequencies in outdoor environment remains a challenging question to be answered.

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