On The Connection between Power-Tail Distributions and Long-Range Dependencies

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1 Introduction

Measurements of recent network data traffic have revealed interesting properties which may have significant consequences to the modeling of broadband ISDN and ATM networked systems. Leland et al. at Bellcore Morristown Research Center have analyzed millions of packets on several Ethernet LAN’s and millions of frame data by Variable-Bit-Rate (VBR) video services [11, 12]. In [11, 12] and in numerous other studies [1, 2, 3, 4, 8, 10] network packet traffic is characterized by high-variability or “burstiness” over a wide range of time scales. In other words, the network traffic looks the same when measured over time intervals ranging from milliseconds to minutes to hours, etc. [5]. Data traffic of this type is said to be self-similar or fractal in nature [11] [12]. Self-similar traffic is very different from both conventional telephone traffic and from the currently accepted norm for models of packet traffic. One reason is that \( r(k|\Delta) \), the autocorrelation function lag-\( k \) of the number of arrivals time interval, \( \Delta \), (the counting process), \( N(t) \), must converge to zero so slowly such that \( \sum_{k=1}^{\infty} r(k|\Delta) = \infty \) [15]. When the aforementioned occurs it is said that the autocorrelation function possesses the property of long-range autocorrelation or long-range dependencies (LRD). Thus, it can be argued that any realistic model of such data traffic should (at the very least) possess one of the following properties: 1) Exhibit self-similar or fractal behavior; 2) Have a counting process which possesses LRD; or 3) An interarrival stream with LRD.

There are traffic models that exist which emulate the aforementioned behaviors. Fractional Brownian Motion (FBM) models have been constructed such that they possess self-similar behavior and long-range dependencies in the counting processes. Consequently, FBM has been used as arrival processes to queues. For example, Norris has effectively utilized FBM to study the performance of storage systems with self-similar input [18]. Other traffic modeling approaches include trace-driven simulations. In trace-driven simulations actual traffic data is obtained from, say, an Ethernet and is used as an interarrival stream to a simulation. Erramilli, Narayan, and Willinger have used this strategy to study the impact of LRD upon single server queues [5]. It is becoming evident that developing amenable queuing models which exhibit properties enumerated by the above would prove very beneficial for studying the performance impact of self-similar data traffic upon communication systems.

One way to engender tractable queuing models that can be solved analytically, is by using power-tail (PT) distributions. A special sub-class of them, called truncated power-tail distributions (\( PT(m) \)’s), were developed by Greiner, Jobmann, and Lipsky [9]. The
$PT(m)$’s are significant because they have a matrix exponential or phase representation; thus, they are amenable for analytic queueing analysis. Furthermore, $PT(m)$’s exhibit LRD in its associated counting process [15]. In the limiting case, that is as $m \to \infty$, the $PT(m)$’s become power-tail distributions [9]. It is important to note that power-tail distributions encompass ALL Lévi-Pareto functions.

In this paper we compute system measurements of an $SM/M/1$ queue where the $SM$ represents a semi-Markov process corresponding to the superpositioning of two interarrival streams. One of the interarrival streams is a Poisson process and the other a $PT(m)$ renewal process. The merged process is referred to as the $PT \otimes \lambda$ arrival stream. This model provides insight for teletraffic systems considered recently in the literature since it was shown in [7] that the $PT \otimes \lambda$ process is self-similar and has LRD in its interarrival stream and argued that it possesses LRD in its counting process. The $SM/M/1$ queue’s performance measurements are compared to that of a $PT(m)/M/1$ system where the arrival process consists of renewal power-tailed interarrivals, and to that of a $GI(SM)/M/1$ system where the correlations in the interarrival stream have been removed. We go on to illustrate that analysis of the $GI(SM)$ case provides analytical confirmation of recent studies which utilize experimental queueing analysis on shuffled traces (i.e. [5]). That is, our results are consistent with that work in the sense of eliminating correlations improves queueing performance. Furthermore, we illustrate the aforementioned comparison cases represent boundaries of a spectrum. The performance of the $PT \otimes \lambda$ process approaches that of the $PT(m)$ case when the amount of Poisson added to the stream is negligible; or, it approaches that of the $GI(SM)$ case when the amount of Poisson added is augmented. Since both bounds are renewal processes, we argue that LRD may be considered as one particular type of power-tailed behavior and that this behavior, in general, should be considered as the more general indicator of potential performance problems.

2 Background

This section presents the background necessary for this paper. The notion of stochastic self-similarity, LAQT, power-tail distributions, and properties of the interarrival processes will be discussed.

2.1 Stochastic Self-Similarity

Stochastic self-similarity is not the same as topological self-similarity. Roughly speaking, topological self-similarity implies that if we examine a small portion, $S'$ of some set $S$, $S'$ resembles the original set $S$. In other words, there exists some exact affine transformation which maps $S'$ directly onto $S$.

Stochastic self-similarity is a weaker version of its topological cousin since there can be no exact affine transformations. However, one way to quantify self-similarity is to realize that there exists some scaling factor, $\kappa$, such that random variables in a self-similar distribution, with parameter, $\alpha$ can be generated in the following manner. Let $A_n$ be the random variable
(r.v.) for the sample average drawn from the original distribution, \( F_\alpha(\cdot) \). In other words,
\[
A_n = \frac{1}{n} \sum_{k=1}^{n} X_k
\]
where \( X = \{X_1, X_2, \cdots, X_n\} \) is a sequence of iid random variables generated from \( F_\alpha(\cdot) \).

Next define the r.v.,
\[
Z_n = n^\kappa [A_n - E(X)] \quad \text{where} \quad \kappa = 1 - \frac{1}{\alpha} \quad \text{for} \quad 1 < \alpha \leq 2. \tag{1}
\]
The theory of \( \alpha \)-stable distributions states that the \( Z_n \)'s are identically distributed. In this context it can be shown that all distributions are asymptotically self-similar. Furthermore note that \( Z_n \) in Equation (1) does not adhere to convergence properties inherent in distributions which obey the Central Limit Theorem (CLT). For instance, random variables which follow the CLT can be generated in the following way:
\[
Z_n = n^\frac{1}{2} [A_n - E(X)], \tag{2}
\]
where again, \( A_n \) represents a sample average drawn from the original distribution, \( F_2(\cdot) \); and, \( A_n = \frac{1}{n} \sum_{k=1}^{n} X_k \) where \( X = \{x_1, x_2, \cdots, x_n\} \) is a sequence of iid random variables generated from \( F_2(\cdot) \).

What Equations (1) and (2) are implying is that self-similar distributions contract much more slowly as compared to distributions which adhere to the CLT. In other words, in general, it takes many more samples from a self-similar distribution for the random variable \( Z_n \) in Equation (1) to converge than it does \( Z_n \) in Equation (2).

### 2.2 The Linear Algebraic View of Queueing Systems

This section discusses how Probability Distribution Functions (PDF) can be represented by vector-matrix pairs. This procedure is then extended to arbitrary sub-systems. By a sub-system we mean any collection of servers and queues, where the total number of customers is not fixed. For instance, a closed loop of two servers and queues is made up of two sub-systems. Furthermore, the single server and the queue of, say, an open \( M/G/1 \) queue is a sub-system. The arrival process is also a sub-system and the two together make up the system.

#### 2.2.1 Representation of Distribution Functions

It is well known that any PDF can be represented arbitrarily closely by an \( m \) dimensional vector-matrix pair, \( \langle \mathbf{p}, \mathbf{B} \rangle \), using the following formulas [17]. Following the notation in [14], let \( X \) be a random variable greater than or equal to 0. Then its PDF is
\[
F(x) = Pr(X \leq x) = 1 - \mathbf{p} \exp(-x\mathbf{B})e'. \tag{3}
\]
Its probability density function (pdf) is
\[
f(x) = \frac{dF(x)}{dx} = \mathbf{p} \exp(-x\mathbf{B}) \mathbf{B} e'. \tag{4}
\]
where \( e' \) is a column \( m \)-vector of all 1’s, and
\[
p e' = \sum_{j=1}^{m} (p)_j = 1. \tag{5}
\]
The linear operator, \( \Psi \), which maps any \( m \times m \) matrix, \( X \), into a scalar is defined by:
\[
\Psi[X] = p X e'. \tag{6}
\]
The Reliability Function is
\[
R(x) = 1 - F(x) = Pr(X > x) = \Psi[\exp(-xB)]. \tag{7}
\]
It can be shown that the \( n^{th} \) moments satisfy:
\[
E(X^n) = \int_{0}^{\infty} x^n f(x) \, dx = n! p V^n e' = n! \Psi[V^n] \tag{8}
\]
where \( V = B^{-1} \). The Laplace Transform of \( f(x) \) is given by:
\[
F^*(s) := \int_{0}^{\infty} e^{-sx} f(x) \, dx = p B [sI + B]^{-1} e' = \Psi[(I + sV)^{-1}]. \tag{9}
\]
Functions which can be represented exactly in this way are called Matrix Exponential (ME) distributions. It should be mentioned that our notation differs from that of Neuts [17] in the following way. Neuts defines an \( m + 1 \) dimensional square matrix, the infinitesimal generator, which compares with LAQT in the following way:
\[
Q = \begin{bmatrix} T & T^0 \\ 0 & 0 \end{bmatrix} \iff \begin{bmatrix} -B & B e' \\ 0 & 0 \end{bmatrix}.
\]
Also, the first \( m \) components of his \((m + 1)\) dimensional initial vector, \( \alpha_0 \), is our vector, \( p \).

### 2.2.2 Matrix Representations of Sub-systems

The equations of the previous section can be extended to any Markovian-like sub-system with a countable state space. Let \( \varphi_0 \) be the probability vector of the state of the sub-system at the time \( x = 0 \), with \( \varphi_0 e' = 1 \) (\( e' \) is the sub-system vector equivalent to \( e' \)), and \( B \) is the infinitesimal generator matrix of the process. Then, as in (7), \( \varphi_0 \exp(-xB)e' \) has the interpretation of the probability that the process has not ended by time \( x \). Furthermore, the \( i^{th} \) component of the vector \( \varphi_0 \exp(-xB) \) has the following meaning.

*Given that the sub-system was in vector state \( \varphi_0 \) at time \( x = 0 \), \( \varphi_0 \exp(-xB)_i \) is the probability that the process has not yet completed by time \( x \), AND the sub-system is in state \( i \).*

Usually, \( B \) can be constructed from the underlying Markov chain using the relation
\[
B = M(I - P), \tag{10}
\]
where $\mathcal{M}$ is a diagonal matrix whose $ii^{th}$ component is the probability rate of leaving state $i$, and $\mathcal{P}$ is a sub-stochastic matrix whose $ij^{th}$ component is the probability that the subsystem will transfer to state $j$ after leaving state $i$. At least one of the row sums of $\mathcal{P}$ (i.e., $\mathcal{P}e'$) is strictly less than 1. Thus, there exists state sequences which result in a departure from the sub-system (often visualized as passage to an absorbing state). The requirement that $[\mathcal{I} - \mathcal{P}]$ be invertible is equivalent to there being an exit path from every state.

2.2.3 Inter-Departure Distributions

This material is covered in detail in [6]. We will summarize the results here. Let $\{X_n \mid n \geq 1\}$ be a set of random variables where $X_n$ denotes the time for the $n^{th}$ process, or interdeparture time of the $n^{th}$ customer. Consider the following matrix.

**Definition $[\mathcal{L}]$:** Given that the sub-system is in state $i$, $[\mathcal{L}]_{ij} \Delta$ is the probability that a departure will occur within the small time interval, $\Delta$, and the sub-system will be in state $j$ immediately afterwards. In other words, $[\mathcal{L}]_{ij}$ is the sub-system instantaneous departure rate from state $i$ which leaves behind state $j$.

From this definition, it follows that $\sum_j \mathcal{L}_{ij} = [\mathcal{L}e']_i$ is the sub-system instantaneous departure rate from state $i$. But that is what $[\mathcal{B}e']_i$ is. Thus,

$$\mathcal{L}e' = \mathcal{B}e'. \tag{11}$$

Although $\mathcal{L}$ and $\mathcal{B}$ are related by this relation, they describe different parts of the process of interest. $\mathcal{B}$ generates what happens before the departure, while $\mathcal{L}$ tells what happens immediately after the departure. (11) states that They agree about the rate of departure.

Let $\mathcal{V} = \mathcal{B}^{-1}$, then let

$$\mathcal{Y} = \mathcal{V}\mathcal{L} \tag{12}$$

where $\mathcal{Y}$ is stochastic since $\mathcal{Y}e' = e'$. Each $ij^{th}$ entry in the $\mathcal{Y}$ matrix represents the probability that after a departure when the system in state $i$, the system goes to state $j$.

2.3 The Power-Tail Interarrival Process

A power-tail distribution will be used as the interarrival process of the $PT(m)/M/1$ queue. It has been argued that these distributions can be used as a model for network traffic behavior [15]. The reason is that long-interarrival times are generated which statistically necessitate large “bursts” of traffic. Power-tail distributions encompass ALL Lévi-Pareto and power-tail-like functions [9]. Power-tails have been shown to be self-similar functions, possess long-range autocorrelations in the counting process, and infinite variance [9] [15]. See [9] and [15] for a more thorough description and analysis. The reliability function, $R(x)$, for any power-tail distribution has the property

$$R(x) \Rightarrow \frac{c}{x^\alpha}, \text{ where } \alpha > 0, c > 0. \tag{13}$$
its pdf, after differentiation, is
\[ f(x) = \frac{\alpha c}{x^{\alpha+1}}. \]  
(14)

Note that the variance does not exist for \( f(x) \) if \( 0 < \alpha \leq 2 \). In general, for all the moments, it can be shown that
\[ E(X^\ell) = \int_0^\infty x^\ell f(x) \, dx = \infty \quad \forall \; \ell \geq \alpha. \]  
(15)

[9] developed a family of functions which emulates the power-tail distribution and can be utilized in LAQT and in analytic models generally. The reliability function for a TPT-\( M \) distribution is given by
\[ R(x) = \frac{1 - \theta}{1 - \theta^M} \sum_{n=0}^{M-1} \theta^n \exp \left(-\frac{\mu x}{\gamma^n}\right), \quad \text{where } 0 < \theta < 1 \text{ and } \gamma > 1. \]  
(16)

If the following limit function is defined
\[ R(x) = \lim_{M \to \infty} R_M(x) = (1 - \theta) \sum_{n=0}^{\infty} \theta^n \exp \left(-\frac{\mu x}{\gamma^n}\right), \]  
(17)

then [9] has shown that \( R(x) \) satisfies (13), and \( \alpha \) is related to \( \theta \) and \( \gamma \) by
\[ \theta \gamma^\alpha = 1, \quad \text{or} \quad \alpha = \frac{\log(\theta)}{\log(\gamma)}. \]  
(18)

Furthermore [9] has shown that all moments are infinite as \( M \to \infty \), that is,
\[ E(X) = \lim_{M \to \infty} E(X^\ell_M) = \infty \quad \forall \; \ell \geq \alpha. \]  
(19)

### 2.4 The \( PT \otimes \lambda \) Interarrival Process

The autocorrelated interarrival process utilized in this paper is a Poisson process superimposed upon a power-tail distribution. The basic idea behind this model is that power-tail distributions tend to have long interarrival times with no arrivals. Unfortunately, actual measured network data traffic (i.e. [12]) does not show this property; in other words, there are no long periods without arrivals. The \( PT \otimes \lambda \) process attempts to overcome this problem by mixing a Poisson process with a power-tail distribution to break up the long interarrival time gaps. It was shown in [7] that this process has long-range autocorrelations in the counting process and in the interarrival times. It should be noted that long-range dependencies have the property that for large \( k \) its autocorrelation function, \( r(k) \), behaves like
\[ r(k) \sim O \left( \frac{c}{k^{\alpha-1}} \right) \quad \text{where } c \text{ is some constant}. \]  
(20)

The \( PT \otimes \lambda \) process can be represented by a Matrix Exponential Distribution or a Phase Distribution. Since matrices can be used to represent the power-tail and Poisson distributions, [6] showed that the state space reduces to that of the power-tail. In other words, we

\[^*\text{The relationship between the Hurst parameter, } H \text{ and } \alpha \text{ is given in [13] by: } H = (3 - \alpha)/2 \quad \text{or} \quad \alpha = 3 - 2 \cdot H.\]
do not need a direct product or Kronecker Product to represent the superimposed state space
\[ B = B + \lambda I, \]  
and
\[ \mathcal{L} = B Q + \lambda I. \]  

3 The \( SM/M/1, PT(m)/M/1, \) and \( GI(SM)/M/1 \) Queues

This section discusses the \( SM/M/1, PT/M/1, \) and \( GI(SM)/M/1 \) queues and addresses means to obtain their performance measurements.

3.1 The \( SM/M/1 \) Queue

Since there are no explicit solutions for the \( SM/M/1 \) queue, we must resort to an iterative technique. One way to obtain performance measurements is to utilize Neut’s algorithmic approach [17]. The generator matrix of this process, \( Q \), is given by
\[
Q = \begin{bmatrix}
-\mathcal{L} & \mathcal{L} & 0 & 0 & \cdots \\
\lambda I & -(B + \lambda I) & \mathcal{L} & 0 & \cdots \\
0 & \lambda I & -(B + \lambda I) & \mathcal{L} & \cdots \\
0 & 0 & \lambda I & -(B + \lambda I) & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\]  

In order to compute the steady-state probabilities the matrix \( R \) must be found that satisfies the following quadratic equation
\[
A_0 + RA_1 + R^2 A_2 = 0,
\]  
where \( A_0 = -\mathcal{L}, A_1 = -(B + \lambda I), \) and \( A_2 = \lambda I \). Generally, the matrix \( R \) is solved via a recursive process. See [17] for one method.

The steady-state probabilities can be obtained by the following equation
\[
r(n) = \pi(I - R)R^k, \text{ for } k \geq 0.
\]  

The \( R \) matrix can be used to compute the system time
\[
\bar{T} = \pi r(1)R(I - R)^{-2} \mathcal{E} \mathcal{F}_1
\]

\[\text{Note that since } B_\lambda \text{ is the service rate matrix for the Poisson process and is 1-dimensional, the Kronecker product } B_{PT} \otimes B_\lambda = B_{PT} \otimes \lambda = B_{PT} + \lambda I \text{ and is the same dimension as the service rate matrix for the power-tail.}\]
where $\bar{\lambda}_1$ is the mean rate of the superimposed interarrival process, and the stationary or residual vector, $\pi$, is

$$\pi = \frac{\psi \psi'}{\psi \psi'}$$

(27)

where $\psi$ satisfies $\psi \psi' = \psi$. In essence, $\psi$ is the steady-state start-up vector. See [6] for a more complete description.

3.2 The $PT/M/1$ Queue

The $PT(m)/M/1$ queue is essentially a $GI/M/1$ queue with the "$PT(m)$" representing a truncated power-tail interarrival process. It is known that the steady-state probability for finding $k$ customers in a $GI/M/1$ queue, $\pi(k)$, is given in [14] by

$$r(0) = 1 - \rho$$
$$r(k) = (1 - s) \cdot \rho s^{k-1}, \quad k > 0$$

(28)

(29)

where $s$ is the geometric parameter satisfying the equation

$$s = B^* [\lambda (1 - s)].$$

(30)

$B^*(s)$ is the Laplace transform of the pdf for the interarrival distribution. Alternatively, in LAQT, $s$ is the smallest eigenvalue of the matrix

$$A = I + \frac{1}{\lambda} B - Q$$

(31)

where $Q = e' p$. Let $\bar{\lambda}$ be the mean interarrival time. Then

$$\rho = \frac{1}{\bar{\lambda} \bar{\lambda}}$$

(32)

and, the mean queue length (including the one being served) for the process is

$$\bar{q} = \sum_{k=0}^{\infty} \pi(k) = \frac{\rho}{1 - s}.$$

(33)

The mean system time of this queue can be obtained by applying Little’s Law

$$\bar{T} = \frac{1}{\bar{\lambda} \bar{\lambda}}.$$

(34)

3.3 The $GI(SM)/M/1$ Queue

The $GI(SM)/M/1$ queue is analytically similar to the $PT(m)/M/1$ queue. The reason is that its interarrival process is uncorrelated, hence, an explicit solution exists. The steady-state probability for finding $k$ customers in the queue, $r(k)$, is given by Equations (28-30), where, now, $s$ is the smallest eigenvalue of the matrix

$$A = I + \frac{1}{\lambda} B - Q$$

(35)
where $Q = e^Q$. By using Little’s Law the mean system time of this queue can be determined by

$$T = \frac{1}{1 - s}. \quad (36)$$

Figure 1: The System Times of the $PT(32)/M/1$, $SM/M/1$, and $GI(SM)/M/1$ Queues. The amount of Poisson added to the $PT \otimes \lambda$ process ranges from 10% to 90%. Note how the performance of the $SM/M/1$ queue approaches that of the $PT(32)/M/1$ queue as the amount of Poisson added to the stream is diminished. Similarly, the performance of the $SM/M/1$ system approaches that of the $GI(SM)/M/1$ queue as the amount of Poisson added is augmented. The mean of the power-tail was fixed to 1.0 while $\lambda$ was varied when generating the $PT \otimes \lambda$ process.
4 Some Results

In this section, we compute the steady-state system times of the $PT(32)/M/1$, $SM/M/1$, and $GI(SM)/M/1$ queues and analyze our results with the ones obtained from [5].

4.1 Queueing Performance

From Figure (1) it is apparent that the $PT(32)/M/1$ queue has the overall worst performance. One way to understand why this is occurring is realize with power-tail distributions, large events will eventually occur. Long interarrival times create “gaps” in the arrival stream which indicate long periods of no arrivals to the system. The consequence of these long periods of no arrivals is that it statistically necessitates intervals where large numbers of arrivals will occur in “bursts” to engender a finite mean. The consequence of these bursts of traffic is that the performance of the system, as a whole, degrades.

Figure (1) also indicates that varying the amount of Poisson which is added to the arrival process affects the performance of the $SM/M/1$ queue. For the $PT(32)$ and $PT \odot \lambda$ streams in Figure (1), when the amount of Poisson added to the process is 10%, the behavior of the queue is very similar to that of the $PT(32)/M/1$ queue. This implies that the $PT \odot \lambda$ interarrival stream has the similar bursty behavior of the power-tail renewal process, however, not quite as severe.

Adding more Poisson to the stream has the effect of subduing the bursty behavior of the $PT \odot \lambda$ process. The reason is that fewer bursts of traffic are needed to engender a finite mean since Poisson arrivals are breaking up long interarrival time gaps generated by the power-tail distribution. Consequently, as more Poisson is added to the stream, performance of this system improves. It should be noted that $SM/M/1$ queueing behavior asymptotically approaches that of the $M/M/1$ system as the percentage of Poisson added to the $PT \odot \lambda$ process approaches 100%.

The $GI(SM)/M/1$ queue uses the same interarrival distribution as the $SM/M/1$ queue; however, the difference is that its arrival stream is uncorrelated. In a sense, one can think of the $PT \odot \lambda$ interarrival stream being generated and then “shuffled” or “scrambled” to create a renewal process. One important notion to consider is why does this system perform better than that of the $PT(32)/M/1$ and $SM/M/1$ queues? Consider the $PT \odot \lambda$ process when the amount of Poisson added is 10%. It is known from [7] that this stream is positively correlated; thus, the covariance function, $\text{Cov}(X, X_{k+})$, at lag-$k$ is positive. This implies, in essence, that smaller interarrival times in the $PT \odot \lambda$ process are followed by larger interarrival times. This behavior has tendency to degrade performance. Shuffling the arrival stream, in effect, removes the aforementioned predisposition; thus, the performance of the system improves.

4.2 Boundaries of a Spectrum

The results of Figure (1) indicate that the $PT(32)/M/1$ and $GI(SM)/M/1$ cases represent boundaries of a spectrum of performance for the $(PT \odot \lambda)/M/1$ queue. That is, as the percentage of the power-tail distribution is augmented, the performance of the $PT \odot \lambda$ queue approaches that of the $PT(32)/M/1$ queue. Alternatively, as the amount of Poisson added...
to the $PT \otimes \lambda$ stream is diminished, its performance approaches that of the $GI(SM)/M/1$ queue. One interesting fact is that both the $PT(32)$ and the $GI(SM)$ interarrival processes are renewal processes. In other words, by definition, these interarrival streams do not possess any correlated behavior. Thus, one conclusion which can be inferred from Figure (1) is that LRD could be viewed as one particular type of power-tailed behavior in systems; and, that power-tailed behavior (i.e. the $PT(32)$ process) should be the more general indication of performance degregation.

4.3 Analytic Confirmation of Trace-Driven Simulations

Recently, Erramilli examined actual arrival data of Ethernet traffic [5]. The data showed LRD in both the interarrival and counting processes. They used this Ethernet trace to simulate arrivals to a deterministic server in order to compute the mean system times of the queue. They subsequently scrambled the trace and performed the same experiment with the shuffled data. It was discovered that the performance of the correlated trace was significantly worse than that of the uncorrelated or shuffled trace. Consequently, they concluded that LRD were a major contributing factor for performance degradation in queueing systems.

Figure (1) indicates that the shuffled trace has the overall best performance when compared to the $SM/M/1$ and $PT(32)/M/1$ systems. Thus, the $SM/M/1$ and $GI(SM)/M/1$ queueing models provide analytic confirmation of results from trace driven simulations regarding arrival streams exhibiting LRD. The reason is that the analytic results agree in principle with the results by Erramilli.

It should be noted that Erramilli’s conclusions imply that LRD have some arcane effect upon performance while the analytic model implies that autocorrelations retain the information of long interarrival times generated by the power-tail distribution hidden by the Poisson arrivals. This notion will be explored further in the subsequent section when we examine the nexus between power-tail distributions and LRD.

4.4 Conclusion

In this paper we compared the steady-state performance of the $PT/M/1$, $SM/M/1$, and $GI(SM)/M/1$ queues where the $SM$ represented a semi-Markov process consisting a Poisson stream superpositioned upon a $PT(32)$ renewal process. The merged process was referred to as the $PT \otimes \lambda$ arrival stream. We argued that this model provides insight for teletraffic systems considered recently in the literature since it was shown in [7] that this process is self-similar and has LRD in its interarrival stream and suggested that it possesses LRD in its counting process. When the $SM/M/1$ queueing performance measurements were compared to that of a $PT(m)/M/1$ system and to that of the $GI(SM)/M/1$ system, where the correlations in the $PT \otimes \lambda$ stream had been removed, it was argued that: 1) This provided analytical confirmation of recent studies which utilize experimental queueing analysis on shuffled traces (i.e. [5]). That is, our results agreed in principle with Erramilli’s study in the sense that eliminating LRD improves queueing performance; and, 2) Illustrated that the $PT(32)/M/1$ and $GI(SM)/M/1$ systems represent boundaries of a performance spectrum. That is, the performance of the $PT \otimes \lambda$ process approaches that of the $PT(m)$ case when amount of Poisson added to the stream is negligible; or, it approaches that of the $GI(SM)$
case when the amount of Poisson added is augmented. Since both bounds are renewal processes, we argued that LRD could be viewed as one type of power-tailed behavior and that this behavior, in general, should be considered as the more general indicator of potential performance problems.

References


