PROBLEM 3.79

If $P = 20$ lb, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

From the solution to Problem 3.78:

16-lb force: $M_1 = -(480 \text{ lb \cdot in.})k$

40-lb force: $M_2 = 8\sqrt{5}[(10 \text{ lb \cdot in.})i + (30 \text{ lb \cdot in.})j + (15 \text{ lb \cdot in.})k]$

$P = 20$ lb $M_3 = r_c \times P$

$= (30 \text{ in.})i \times (20 \text{ lb})k$

$= (600 \text{ lb \cdot in.})j$

$M = M_1 + M_2 + M_3$

$= -(480)k + 8\sqrt{5}(10i + 30j + 15k) + 600j$

$= (178.885 \text{ lb \cdot in.})i + (1136.66 \text{ lb \cdot in.})j - (211.67 \text{ lb \cdot in.})k$

$M = \sqrt{(178.885)^2 + (1136.66)^2 + (211.67)^2}$

$= 1169.96 \text{ lb \cdot in.}$

$M = 1170 \text{ lb \cdot in.}$

$\lambda_{axis} = \frac{M}{M} = 0.152898i + 0.97154j - 0.180921k$

$\cos \theta_x = 0.152898$

$\cos \theta_y = 0.97154$

$\cos \theta_z = -0.180921$

$\theta_x = 81.2^\circ$, $\theta_y = 13.70^\circ$, $\theta_z = 100.4^\circ$
PROBLEM 3.87

Three control rods attached to a lever \( ABC \) exert on it the forces shown. (a) Replace the three forces with an equivalent force-couple system at \( B \). (b) Determine the single force that is equivalent to the force-couple system obtained in part a, and specify its point of application on the lever.

SOLUTION

(a) First note that the two 90-N forces form a couple. Then

\[ F = 216 \text{ N} \angle \theta \]

where

\[ \theta = 180^\circ - (60^\circ + 55^\circ) = 65^\circ \]

and

\[ M = \Sigma M_B \]

\[ = (0.450 \text{ m})(216 \text{ N})\cos55^\circ - (1.050 \text{ m})(90 \text{ N})\cos20^\circ \]

\[ = -33.049 \text{ N} \cdot \text{m} \]

The equivalent force-couple system at \( B \) is

\[ F = 216 \text{ N} \angle 65.0^\circ; \quad M = 33.0 \text{ N} \cdot \text{m} \]

(b) The single equivalent force \( F' \) is equal to \( F \). Further, since the sense of \( M \) is clockwise, \( F' \) must be applied between \( A \) and \( B \). For equivalence,

\[ \Sigma M_B: \quad M = aF' \cos55^\circ \]

where \( a \) is the distance from \( B \) to the point of application of \( F' \). Then

\[ -33.049 \text{ N} \cdot \text{m} = -a(216 \text{ N})\cos55^\circ \]

\[ a = 0.26676 \text{ m} \]

or

\[ F' = 216 \text{ N} \angle 65.0^\circ \text{ applied to the lever 267 mm to the left of } B \]
PROBLEM 3.93

An antenna is guyed by three cables as shown. Knowing that the tension in cable $AB$ is 288 lb, replace the force exerted at $A$ by cable $AB$ with an equivalent force-couple system at the center $O$ of the base of the antenna.

SOLUTION

We have

$$d_{AB} = \sqrt{(-64)^2 + (-128)^2 + (16)^2} = 144 \text{ ft}$$

Then

$$\mathbf{T}_{AB} = \frac{288 \text{ lb}}{144} (-64\mathbf{i} - 128\mathbf{j} + 16\mathbf{k})$$

$$= (32 \text{ lb})(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k})$$

Now

$$\mathbf{M} = \mathbf{r}_{A/O} \times \mathbf{T}_{AB}$$

$$= 128\mathbf{j} \times 32(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k})$$

$$= (4096 \text{ lb} \cdot \text{ft})\mathbf{i} + (16,384 \text{ lb} \cdot \text{ft})\mathbf{k}$$

The equivalent force-couple system at $O$ is

$$\mathbf{F}' = -(128.0 \text{ lb})\mathbf{i} - (256 \text{ lb})\mathbf{j} + (32.0 \text{ lb})\mathbf{k}$$

$$\mathbf{M} = (4.10 \text{ kip} \cdot \text{ft})\mathbf{i} + (16.38 \text{ kip} \cdot \text{ft})\mathbf{k}$$