PROBLEM 4.104

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the weight and location of the lightest block that should be placed on the plate if the tensions in the three wires are to be equal.

SOLUTION

Free-Body Diagram:

Let \(-W_b\) be the weight of the block and \(x\) and \(z\) the block’s coordinates.

Since tensions in wires are equal, let

\[ T_A = T_B = T_C = T \]

\[ \sum M_0 = 0: \quad (r_A \times T\hat{j}) + (r_B \times T\hat{j}) + (r_C \times T\hat{j}) + r_c \times (-W\hat{j}) + (x\hat{i} + z\hat{k}) \times (-W_b\hat{j}) = 0 \]

or

\[ (75\hat{k}) \times T\hat{j} + (15\hat{k}) \times T\hat{j} + (60\hat{i} + 30\hat{k}) \times T\hat{j} + (30\hat{i} + 45\hat{k}) \times (-W\hat{j}) + (x\hat{i} + z\hat{k}) \times (-W_b\hat{j}) = 0 \]

or

\[ -75T\hat{i} - 15T\hat{i} + 60T\hat{k} - 30T\hat{i} - 30W\hat{k} + 45W\hat{i} - W_b\hat{x} + W_b\hat{z} = 0 \]

Equate coefficients of unit vectors to zero:

\[ \hat{i}: \quad -120T + 45W + W_bz = 0 \]  \hspace{1cm} (1)

\[ \hat{k}: \quad 60T - 30W - W_bx = 0 \]  \hspace{1cm} (2)

Also,

\[ \sum F_y = 0: \quad 3T - W - W_b = 0 \]  \hspace{1cm} (3)

Eq. (1) + 40 Eq. (3):

\[ 5W + (z - 40)W_b = 0 \]  \hspace{1cm} (4)

Eq. (2) − 20 Eq. (3):

\[ -10W - (x - 20)W_b = 0 \]  \hspace{1cm} (5)
Solving Eqs. (4) and (5) for \( \frac{W_b}{W} \) and recalling that \( 0 \leq x \leq 60 \text{ in.} \), \( 0 \leq z \leq 90 \text{ in.} \),

Eq. (4):
\[
\frac{W_b}{W} = \frac{5}{40 - z} \geq \frac{5}{40 - 0} = 0.125
\]

Eq. (5):
\[
\frac{W_b}{W} = \frac{10}{20 - x} \geq \frac{10}{20 - 0} = 0.5
\]

Thus, \( (W_b)_{\text{min}} = 0.5W = 0.5(80) = 40 \text{ lb} \) \( (W_b)_{\text{min}} = 40.0 \text{ lb} \)

Making \( W_b = 0.5W \) in Eqs. (4) and (5):
\[
5W + (z - 40)(0.5W) = 0 \quad \Rightarrow \quad z = 30.0 \text{ in.}
\]
\[
-10W - (x - 20)(0.5W) = 0 \quad \Rightarrow \quad x = 0 \text{ in.}
\]
PROBLEM 4.116

Solve Problem 4.115, assuming that cable \( EF \) is replaced by a cable attached at points \( E \) and \( H \).

PROBLEM 4.115

The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at \( A \) and \( B \) and by cable \( EF \). Assuming that the hinge at \( B \) does not exert any axial thrust, determine \((a)\) the tension in the cable, \((b)\) the reactions at \( A \) and \( B \).

SOLUTION

\[
\begin{align*}
\mathbf{r}_{BA} &= (38-8)i = 30i \\
\mathbf{r}_{EA} &= (30-4)i + 20k = 26i + 20k \\
\mathbf{r}_{GA} &= \frac{38}{2}i + 10k = 19i + 10k \\
\overline{EH} &= -30i + 12j - 20k \quad EH = 38 \text{ in.} \\
T &= T \frac{EH}{EH} = T \frac{30i + 12j - 20k}{38} \\
\Sigma M_A = 0: \quad \mathbf{r}_{EA} \times T + \mathbf{r}_{GA} \times (-75j) + \mathbf{r}_{BA} \times \mathbf{B} = 0
\end{align*}
\]

Coefficient of \( i \):
\[
-(12)(20) \frac{T}{38} + 750 = 0 \quad T = \frac{118.75}{38} \Rightarrow T = 118.8 lb
\]

Coefficient of \( j \):
\[
-(-600 + 520) \frac{118.75}{38} - 30B_z = 0 \quad B_z = \frac{8.33}{38} \text{ lb}
\]

Coefficient of \( k \):
\[
(26)(12) \frac{118.75}{38} - 1425 + 30B_y = 0 \quad B_y = \frac{15.00}{38} \text{ lb} \quad B = (15.00 \text{ lb})j - (8.33 \text{ lb})k
\]
PROBLEM 4.116 (Continued)

\[ \sum \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0 \]

Coefficient of \( \mathbf{i} \):
\[ A_x - \frac{30}{38} (118.75) = 0 \quad A_x = 93.75 \text{ lb} \]

Coefficient of \( \mathbf{j} \):
\[ A_y + 15 + \frac{12}{38} (118.75) - 75 = 0 \quad A_y = 22.5 \text{ lb} \]

Coefficient of \( \mathbf{k} \):
\[ A_z - 8.33 - \frac{20}{38} (118.75) = 0 \quad A_z = 70.83 \text{ lb} \]

\[ \mathbf{A} = (93.8 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} + (70.8 \text{ lb})\mathbf{k} \]
PROBLEM 4.120

Solve Problem 4.118, assuming that the bearing at D is removed and that the bearing at C can exert couples about axes parallel to the y and z axes.

PROBLEM 4.118

The bent rod ABEF is supported by bearings at C and D and by wire AH. Knowing that portion AB of the rod is 250 mm long, determine (a) the tension in wire AH, (b) the reactions at C and D. Assume that the bearing at D does not exert any axial thrust.

SOLUTION

Free-Body Diagram: \( \Delta A BH \) is equilateral.

Dimensions in mm

\[
\begin{align*}
\mathbf{r}_{HC} &= -50\mathbf{i} + 250\mathbf{j} \\
\mathbf{r}_{FC} &= 350\mathbf{i} + 250\mathbf{k} \\
T &= T(\sin 30^\circ)\mathbf{j} - T(\cos 30^\circ)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k}) \\
\Sigma M_C &= 0: \mathbf{r}_{FC} \times (-400\mathbf{j}) + \mathbf{r}_{HC} \times T + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0
\end{align*}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
i & j & k & i & j & k \\
\hline
350 & 0 & 250 & -50 & 250 & 0 & T + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0 \\
0 & -400 & 0 & 0 & 0.5 & -0.866 \\
\hline
\end{array}
\]

Coefficient of \( \mathbf{i} \): \(+100 \times 10^3 - 216.5T = 0 \quad T = 461.9 \, \text{N}\)

Coefficient of \( \mathbf{j} \): \(-43.3(461.9) + (M_C)_y = 0 \)

\( (M_C)_y = 20 \times 10^3 \, \text{N} \cdot \text{mm} \)

\( (M_C)_z = 20.0 \, \text{N} \cdot \text{m} \)
PROBLEM 4.120 (Continued)

Coefficient of \( k \): \[-140 \times 10^3 - 25(461.9) + (M_C)_z = 0\]

\[(M_C)_z = 151.54 \times 10^3 \text{ N} \cdot \text{mm}\]

\[(M_C)_z = 151.5 \text{ N} \cdot \text{m}\]

\[\Sigma F = 0: \ C + T - 400 j = 0\]

\[M_C = (20.0 \text{ N} \cdot \text{m})j + (151.5 \text{ N} \cdot \text{m})k \]

Coefficient of \( i \):

\[C_x = 0\]

Coefficient of \( j \):

\[C_y + 0.5(461.9) - 400 = 0 \quad C_y = 169.1 \text{ N}\]

Coefficient of \( k \):

\[C_z - 0.866(461.9) = 0 \quad C_z = 400 \text{ N}\]

\[C = (169.1 \text{ N})j + (400 \text{ N})k\]
PROBLEM 4.144

A lever $AB$ is hinged at $C$ and attached to a control cable at $A$. If the lever is subjected to a 500-N horizontal force at $B$, determine
(a) the tension in the cable, (b) the reaction at $C$.

SOLUTION

Triangle $ACD$ is isosceles with $\angle C = 90^\circ + 30^\circ = 120^\circ$, $\angle A = \angle D = \frac{1}{2}(180^\circ - 120^\circ) = 30^\circ$.

Thus, $DA$ forms angle of $60^\circ$ with the horizontal axis.

(a) We resolve $F_{AD}$ into components along $AB$ and perpendicular to $AB$.

\[ \sum M_C = 0: \quad (F_{AD} \sin 30^\circ)(250 \text{ mm}) - (500 \text{ N})(100 \text{ mm}) = 0 \quad \Rightarrow \quad F_{AD} = 400 \text{ N} \]

(b) $\sum F_x = 0: \quad -(400 \text{ N}) \cos 60^\circ + C_x - 500 \text{ N} = 0 \quad \Rightarrow \quad C_x = +300 \text{ N}$

$\sum F_y = 0: \quad -(400 \text{ N}) \sin 60^\circ + C_y = 0 \quad \Rightarrow \quad C_y = +346.4 \text{ N}$

$C = 458 \text{ N} \angle 49.1^\circ$
PROBLEM 4.146

Two slots have been cut in plate DEF, and the plate has been placed so that the slots fit two fixed, frictionless pins A and B. Knowing that \( P = 15 \text{ lb} \), determine \( (a) \) the force each pin exerts on the plate, \( (b) \) the reaction at F.

SOLUTION

Free-Body Diagram:

\[
\begin{align*}
(a) \quad & \quad \sum F_x = 0: \quad 15 \text{ lb} - B \sin 30^\circ = 0 \quad B = 30.0 \text{ lb } \uparrow 60.0^\circ \uparrow \\
(b) \quad & \quad \sum M_A = 0: \quad -(30 \text{ lb})(4 \text{ in.}) + B \sin 30^\circ(3 \text{ in.}) + B \cos 30^\circ(11 \text{ in.}) - F(13 \text{ in.}) = 0 \\
& \quad -120 \text{ lb}\cdot \text{in.} + (30 \text{ lb}) \sin 30^\circ(3 \text{ in.}) + (30 \text{ lb}) \cos 30^\circ(11 \text{ in.}) - F(13 \text{ in.}) = 0 \\
& \quad F = +16.2145 \text{ lb} \\
(a) \quad & \quad \sum F_y = 0: \quad A - 30 \text{ lb} + B \cos 30^\circ - F = 0 \\
& \quad A - 30 \text{ lb} + (30 \text{ lb}) \cos 30^\circ - 16.2145 \text{ lb} = 0 \\
& \quad A = +20.23 \text{ lb} \\
\end{align*}
\]