PROBLEM 7.37

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam

\[ \sum M_A = 0: \quad E(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (12 \text{ kips})(4 \text{ ft}) - (4.5 \text{ kips})(8 \text{ ft}) = 0 \]

\[ E = +16 \text{ kips} \]

\[ \sum F_y = 0: \quad A_y = 0 \]

\[ \sum F_y = 0: \quad A_y + 16 \text{ kips} - 6 \text{ kips} - 12 \text{ kips} - 4.5 \text{ kips} = 0 \]

\[ A_y = +6.50 \text{ kips} \]

(a) Shear and bending moment

Just to the right of A:

\[ V_1 = +6.50 \text{ kips} \quad M_1 = 0 \]

Just to the right of C:

\[ \sum F_y = 0: \quad 6.50 \text{ kips} - 6 \text{ kips} - V_2 = 0 \]

\[ V_2 = +0.50 \text{ kips} \]

\[ \sum M_2 = 0: \quad M_2 - (6.50 \text{ kips})(2 \text{ ft}) = 0 \]

\[ M_2 = +13 \text{ kip-ft} \]

Just to the right of D:

\[ \sum F_y = 0: \quad 6.50 - 6 - 12 - V_3 = 0 \]

\[ V_3 = +11.5 \text{ kips} \]

\[ \sum M_3 = 0: \quad M_3 - (6.50)(4) - (6)(2) = 0 \]

\[ M_3 = +14 \text{ kip-ft} \]
PROBLEM 7.37 (Continued)

Just to the right of $E$:  

\[ + \sum F_y = 0: \quad V_A - 4.5 = 0 \quad V_A = +4.5 \text{ kips} \]

\[ + \sum M_A = 0: \quad -M_A - (4.5)2 = 0 \quad M_A = -9 \text{ kip} \cdot \text{ft} \]

At $B$:  

\[ V_B = M_B = 0 \]

\[ |V|_{\text{max}} = 11.50 \text{ kips} \]

\[ |M|_{\text{max}} = 14.00 \text{ kip} \cdot \text{ft} \]
PROBLEM 7.39

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam

\[ \sum M_A = 0: \quad B(5 \, \text{m}) - (60 \, \text{kN})(2 \, \text{m}) - (50 \, \text{kN})(4 \, \text{m}) = 0 \]

\[ B = 64.0 \, \text{kN} \]

\[ \sum F_x = 0: \quad A_x = 0 \]

\[ \sum F_y = 0: \quad A_y + 64.0 \, \text{kN} - 6.0 \, \text{kN} - 50 \, \text{kN} = 0 \]

\[ A_y = 46.0 \, \text{kN} \]

(a) Shear and bending-moment diagrams.

From A to C:

\[ \sum F_y = 0: \quad 46 - V = 0 \]

\[ V = 46 \, \text{kN} \]

\[ \sum M_y = 0: \quad M - 46x = 0 \]

\[ M = (46x) \, \text{kN} \cdot \text{m} \]

From C to D:

\[ \sum F_y = 0: \quad 46 - 60 - V = 0 \]

\[ V = -14 \, \text{kN} \]

\[ \sum M_y = 0: \quad M - 46x + 60(x - 2) = 0 \]

\[ M = (120 - 14x) \, \text{kN} \cdot \text{m} \]

For \( x = 2 \, \text{m} \):

\[ M_C = 92.0 \, \text{kN} \cdot \text{m} \]

For \( x = 3 \, \text{m} \):

\[ M_D = 78.0 \, \text{kN} \cdot \text{m} \]
PROBLEM 7.62*

In order to reduce the bending moment in the cantilever beam \( AB \), a cable and counterweight are permanently attached at end \( B \). Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of \( |M|_{\text{max}} \). Consider (a) the case when the distributed load is permanently applied to the beam, (b) the more general case when the distributed load may either be applied or removed.

SOLUTION

\( M \) due to distributed load:

\[
\sum M = 0: \quad -M - \frac{x}{2}w = 0
\]

\[ M = -\frac{1}{2}wx^2 \]

\( M \) due to counter weight:

\[
\sum M = 0: \quad -M + xw = 0
\]

\[ M = wx \]

(a) Both applied:

\[
M = W_x - \frac{w}{2}x^2
\]

\[
\frac{dM}{dx} = W - w = 0 \quad \text{at} \quad x = \frac{W}{w}
\]

And here \( M = \frac{W^2}{2w} > 0 \) so \( M_{\text{max}}, M_{\text{min}} \) must be at \( x = L \)

So \( M_{\text{min}} = WL - \frac{1}{2}wL^2 \). For minimum \( |M|_{\text{max}} \), set \( M_{\text{max}} = -M_{\text{min}}, \)

so

\[
\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \quad \text{or} \quad W^2 + 2wLW - w^2L^2 = 0
\]

\[ W = -wL \pm \sqrt{2w^2L^2} \quad (\text{need}+) \]

\[ W = (\sqrt{2} - 1)wL = 0.414 \ wL \]
PROBLEM 7.62* (Continued)

(b) \( w \) may be removed

\[
M_{\text{max}} = \frac{W^2}{2w} = \left(\frac{\sqrt{2}}{2} - 1\right)^2 wL^2
\]

\[M_{\text{max}} = 0.0858 \, wL^2 \uparrow\]

Without \( w \),
\[M = Wx \]
\[M_{\text{max}} = WL \text{ at } A\]

With \( w \) (see Part a)
\[M = Wx - \frac{w}{2} x^2\]

\[
M_{\text{max}} = \frac{W^2}{2w} \text{ at } x = \frac{W}{w}
\]

\[M_{\text{min}} = WL - \frac{1}{2} wL^2 \text{ at } x = L\]

For minimum \( M_{\text{max}} \), set \( M_{\text{max}} \) (no \( w \)) = \(-M_{\text{min}} \) (with \( w \))

\[WL = -WL + \frac{1}{2} wL^2 \rightarrow W = \frac{1}{4} wL \rightarrow \]

\[M_{\text{max}} = \frac{1}{4} wL^2 \uparrow\]

With
\[W = \frac{1}{4} wL \uparrow\]
PROBLEM 7.68

Using the method of Section 7.6, solve Problem 7.34.

PROBLEM 7.34 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam

\[ \sum F_y = 0: \quad B - P = 0 \]

\[ B = P \]

\[ \sum M_B = 0: \quad M_B - M_0 + PL = 0 \]

\[ M_B = 0 \]

Shear diagram

At A:

\[ V_A = -P \]

\[ |V|_{\text{max}} = P \]

Bending-moment diagram

At A:

\[ M_A = M_0 = PL \]

\[ |M|_{\text{max}} = PL \]