PROBLEM 13.7

Determine the maximum theoretical speed that may be achieved over a distance of 110 m by a car starting from rest assuming there is no slipping. The coefficient of static friction between the tires and pavement is 0.75, and 60 percent of the weight of the car is distributed over its front wheels and 40 percent over its rear wheels. Assume (a) front-wheel drive, (b) rear-wheel drive.

SOLUTION

Let \( W \) be the weight and \( m \) the mass.

\[ W = mg \]

(a) Front wheel drive:

\[ N = 0.60W = 0.60mg \]

\( \mu_s = 0.75 \)

Maximum friction force without slipping:

\[ F = \mu_s N = (0.75)(0.60W) = 0.45mg \]

\[ U_{1\rightarrow2} = Fd = 0.45mgd \]

\[ T_1 = 0, \quad T_2 = \frac{1}{2}mv_2^2 \]

Principle of work and energy:

\[ T_1 + U_{1\rightarrow2} = T_2 \]

\[ 0 + 0.45mgd = \frac{1}{2}mv_2^2 \]

\[ v_2^2 = (2)(0.45gd) = (2)(0.45)(9.81 \text{ m/s}^2)(110 \text{ m}) = 971.19 \text{ m}^2/\text{s}^2 \]

\[ v_2 = 31.164 \text{ m/s} \]

\[ v_2 = 112.2 \text{ km/h} \]

(b) Rear wheel drive:

\[ N = 0.40W = 0.40mg \]

\( \mu_s = 0.75 \)

Maximum friction force without slipping:

\[ F = \mu_s N = (0.75)(0.40W) = 0.30mg \]

\[ U_{1\rightarrow2} = Fd = 0.30mgd \]

\[ T_1 = 0, \quad T_2 = \frac{1}{2}mv_2^2 \]

Principle of work and energy:

\[ T_1 + U_{1\rightarrow2} = T_2 \]

\[ 0 + 0.30mgd = \frac{1}{2}mv_2^2 \]

\[ v_2^2 = (2)(0.30gd) = (2)(0.30)(9.81 \text{ m/s}^2)(110 \text{ m}) = 647.46 \text{ m}^2/\text{s}^2 \]

\[ v_2 = 25.445 \text{ m/s} \]

\[ v_2 = 91.6 \text{ km/h} \]

Note: The car is treated as a particle in this problem. The weight distribution is assumed to be the same for static and dynamic conditions. Compare with sample Problem 16.1 where the vehicle is treated as a rigid body.
PROBLEM 13.18

The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars A, causing it to slide on the track, but are not applied on the wheels of cars A or B. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (a) the distance required to bring the train to a stop, (b) the force in each coupling.

SOLUTION

(a) Entire train:

\[ F_A = \mu N_A = (0.35)(80 \text{ kips}) = 28 \text{ kips} \]

\[ v_1 = 30 \text{ mi/h} = 44 \text{ ft/s} \leftarrow \]

\[ v_2 = 0 \quad T_2 = 0 \]

\[ T_1 + V_{1-2} = T_2 \]

\[ \frac{1}{2} \left( \frac{80 \text{ kips} + 100 \text{ kips} + 80 \text{ kips}}{32.2 \text{ ft/s}^2} \right)(44 \text{ ft/s})^2 - (28 \text{ kips})(279.1 \text{ ft}) = 0 \]

\[ x = 279.1 \text{ ft} \]

\[ x = 279 \text{ ft} \]

(b) Force in each coupling:

Car A: Assume \( F_{AB} \) to be in tension

\[ T_1 + V_{1-2} = T_2 \]

\[ \frac{1}{2} \left( \frac{80 \text{ kips}}{32.2 \text{ ft/s}^2} \right)(44 \text{ ft/s})^2 - (28 \text{ kips} + F_{AB})(279.1 \text{ ft}) = 0 \]

\[ 28 \text{ kips} + F_{AB} = +8.62 \text{ kips} \]

\[ F_{AB} = 19.38 \text{ kips (compression)} \]

Car C:

\[ T_1 + V_{1-2} = T_2 \]

\[ \frac{1}{2} \left( \frac{80 \text{ kips}}{32.2 \text{ ft/s}^2} \right)(44 \text{ ft/s})^2 + F_{BC}(279.1 \text{ ft}) = 0 \]

\[ F_{BC} = -8.617 \text{ kips} \]

\[ F_{BC} = 8.62 \text{ kips (compression)} \]
PROBLEM 13.26

A 3-kg block rests on top of a 2-kg block supported by, but not attached to, a spring of constant 40 N/m. The upper block is suddenly removed. Determine (a) the maximum speed reached by the 2-kg block, (b) the maximum height reached by the 2-kg block.

SOLUTION

Call blocks $A$ and $B$. 
\[ m_A = 2 \text{ kg}, \quad m_B = 3 \text{ kg} \]

(a) Position 1: Block $B$ has just been removed.

Spring force:
\[ F_S = -(m_A + m_B)g = -kx \]

Spring stretch:
\[ x_1 = \frac{(m_A + m_B)g}{k} = \frac{(5 \text{ kg})(9.81 \text{ m/s}^2)}{40 \text{ N/m}} = -1.22625 \text{ m} \]

Let position 2 be a later position while the spring still contacts block $A$.

Work of the force exerted by the spring:
\[ (U_{1\to2})_e = -\int_{x_1}^{x_2} kx \; dx \]
\[ = -\frac{1}{2} kx^2 \bigg|_{x_1}^{x_2} = \frac{1}{2} kx_1^2 - \frac{1}{2} kx_2^2 \]
\[ = \frac{1}{2} (40)(-1.22625)^2 - \frac{1}{2} (40)x_2^2 = 30.074 - 20x_2^2 \]

Work of the gravitational force:
\[ (U_{1\to2})_g = -m_Ag(x_2 - x_1) \]
\[ = -(2)(9.81)(x_2 + 1.22625) = -19.62x_2 - 24.059 \]

Total work:
\[ U_{1\to2} = -20x_2^2 + 19.62x_2 + 6.015 \]

Kinetic energies:
\[ T_1 = 0 \]
\[ T_2 = \frac{1}{2} m_Av_2^2 = \frac{1}{2} (2)v_2^2 = v_2^2 \]

Principle of work and energy:
\[ T_1 + U_{1\to2} = T_2 \]
\[ 0 + 20x_2^2 - 19.62x_2 + 6.015 = v_2^2 \]

Speed squared:
\[ v_2^2 = -20x_2^2 - 19.62x_2 + 6.015 \]

At maximum speed,
\[ \frac{dv_2}{dx_2} = 0 \]


**PROBLEM 13.26 (Continued)**

Differentiating Eq. (1), and setting equal to zero,

\[ 2v_2 \frac{dv_2}{dx} = -40x_2 = -19.62 = 0 \]

\[ x_2 = \frac{-19.62}{40} = -0.4905 \text{ m} \]

Substituting into Eq. (1),

\[ v_2^2 = -(20)(-0.4905)^2 - (19.62)(-0.4905) + 6.015 = 10.827 \text{ m}^2/\text{s}^2 \]

Maximum speed:

\[ v^2 = 3.29 \text{ m/s} \]

(b) Position 3: Block A reaches maximum height. Assume that the block has separated from the spring. Spring force is zero at separation.

Work of the force exerted by the spring:

\[ (U_{1\to3})_e = \int_{x_1}^{0} kxdx = \frac{1}{2} kx_1^2 = \frac{1}{2} (40)(1.22625)^2 = 30.074 \text{ J} \]

Work of the gravitational force:

\[ (U_{1\to3})_g = -m_Agh = -(2)(9.81)h = -19.62h \]

Total work:

\[ U_{1\to3} = 30.074 - 19.62h \]

At maximum height,

\[ v_3 = 0, \quad T_3 = 0 \]

Principle of work and energy:

\[ T_1 + U_{1\to3} = T_3 \]

\[ 0 + 30.074 - 19.62h = 0 \]

Maximum height:

\[ h = 1.533 \text{ m} \]
PROBLEM 13.58

A 3-lb collar is attached to a spring and slides without friction along a circular rod in a horizontal plane. The spring has an undeformed length of 7 in. and a constant $k = 1.5 \text{ lb/in.}$ Knowing that the collar is in equilibrium at $A$ and is given a slight push to get it moving, determine the velocity of the collar ($a$) as it passes through $B$, ($b$) as it passes through $C$.

SOLUTION

\[ L_0 = 7 \text{ in.}, \quad L_{DA} = 20 \text{ in.} \]
\[ L_{DB} = \sqrt{(8 + 6)^2 + 6^2} = 15.23 \text{ in.} \]
\[ L_{DC} = 8 \text{ in.} \]
\[ \Delta L_{DA} = 20 - 7 = 13 \text{ in.} \]
\[ \Delta L_{DB} = 15.23 - 7 = 8.23 \text{ in.} \]
\[ \Delta L_{DC} = 8 - 7 = 1 \text{ in.} \]

\[(a) \]
\[ T_A = 0, \quad V_A = \frac{1}{2} k (\Delta L_{DA})^2 = \frac{1}{2} (1.5)(13)^2 = 126.75 \text{ lb-in.} \]
\[ = 10.5625 \text{ lb-ft} \]
\[ T_B = \frac{1}{2} m v_B^2 = \frac{1.5}{g} v_B^2 \]
\[ V_B = \frac{1}{2} (1.5)(8.23)^2 = 50.8 \text{ lb-in.} = 4.233 \text{ lb-ft} \]
\[ T_A + V_A = T_B + V_B: \quad 0 + 10.5625 = \frac{1.5 v_B^2}{32.2} + 4.233 \]
\[ v_B = 11.66 \text{ ft/s} \]
PROBLEM 13.58 (Continued)

(b) 

\[ T_A = 0, \quad V_A = 10.5625 \text{ lb}\cdot\text{ft}, \quad T_C = \frac{1.5}{32.2} v_C^2 \]

\[ V_C = \frac{1}{2} (1.5)(1)^2 = 0.75 \text{ lb}\cdot\text{in.} = 0.0625 \text{ lb}\cdot\text{ft} \]

\[ T_A + V_A = T_C + V_C:\quad 0 + 10.5625 = \frac{1.5}{32.2} v_C^2 + 0.0625 \quad v_C = 15.01 \text{ ft/s} \]
PROBLEM 13.65

A 1-kg collar can slide along the rod shown. It is attached to an elastic cord anchored at $F$, which has an undeformed length of 250 mm and a spring constant of 75 N/m. Knowing that the collar is released from rest at $A$ and neglecting friction, determine the speed of the collar ($a$) at $B$, ($b$) at $E$.

SOLUTION

$L_{AF} = \sqrt{(0.5)^2 + (0.4)^2 + (0.3)^2}$
$L_{AF} = 0.70711 \text{ m}$

$L_{BF} = \sqrt{(0.4)^2 + (0.3)^2}$
$L_{BF} = 0.5 \text{ m}$

$L_{FE} = \sqrt{(0.5)^2 + (0.3)^2}$
$L_{FE} = 0.58309 \text{ m}$

$V = V_e + V_g$

(a) Speed at $B$:

$v_A = 0, \quad T_A = 0$

Point $A$:

$(V_A)_e = \frac{1}{2} k (\Delta L_{AF})^2$
$\Delta L_{AF} = L_{AF} - L_0 = 0.70711 - 0.25$
$\Delta L_{AF} = 0.45711 \text{ m}$

$(V_A)_e = \frac{1}{2} (75 \text{ N/m})(0.45711 \text{ m})^2$
$(V_A)_e = 7.8355 \text{ N} \cdot \text{m}$

$(V_A)_g = (mg)(0.4)$
$= (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0.4 \text{ m})$
$= 3.9240 \text{ N} \cdot \text{m}$

$V_A = (V_A)_e + (V_A)_g$
$= 7.8355 + 3.9240$
$= 11.7595 \text{ N} \cdot \text{m}$
PROBLEM 13.65 (Continued)

Point B:

\[ T_B = \frac{1}{2} mv_B^2 = \frac{1}{2} (1.0 \text{ kg})v_B^2 \]

\[ T_B = 0.5v_B^2 \]

\[(V_B)_e = \frac{1}{2} k(\Delta L_{BF})^2 \]

\[ \Delta L_{BF} = L_{BF} - L_0 = 0.5 - 0.25 \]

\[ \Delta L_{BF} = 0.25 \text{ m} \]

\[(V_B)_e = \frac{1}{2} (75 \text{ N/m})(0.25 \text{ m})^2 = 2.3438 \text{ N} \cdot \text{m} \]

\[(V_B)_s = (mg)(0.4) = (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0.4 \text{ m}) = 3.9240 \text{ N} \cdot \text{m} \]

\[ V_B = (v_B)_e + (V_B)_s = 2.3438 + 3.9240 = 6.2678 \text{ N} \cdot \text{m} \]

\[ T_A + V_A = T_B + V_B \]

\[ 0 + 11.7595 = 0.5v_B^2 + 6.2678 \]

\[ v_B^2 = (5.49169)/(0.5) \]

\[ v_B^2 = 10.983 \text{ m}^2/\text{s}^2 \]

\[ v_B = 3.31 \text{ m/s} \]

(b) Speed at E:

Point A:

\[ T_A = 0 \quad V_A = 11.7595 \text{ N} \cdot \text{m} \quad \text{(from part (a))} \]

Point E:

\[ T_E = \frac{1}{2} mv_E^2 = \frac{1}{2} (1.0 \text{ kg})v_E^2 = 0.5v_E^2 \]

\[(V_E)_e = \frac{1}{2} k(\Delta L_{FE})^2 \]

\[ \Delta L_{FE} = L_{FE} - L_0 = 0.5831 - 0.25 \]

\[ \Delta L_{FE} = 0.3331 \text{ m} \]

\[(V_E)_e = \frac{1}{2} (75 \text{ N/m})(0.3331 \text{ m})^2 = 4.1607 \text{ N} \cdot \text{m} \]

\[(V_E)_s = 0 \quad V_E = 4.1607 \text{ N} \cdot \text{m} \]

\[ T_A + V_A = T_E + V_E \]

\[ 0 + 11.7595 = 0.5v_E^2 + 4.1607 \]

\[ v_E^2 = 7.5988/0.5 \]

\[ v_E^2 = 15.1976 \text{ m}^2/\text{s}^2 \]

\[ v_E = 3.90 \text{ m/s} \]
**PROBLEM 13.72**

A 1-lb collar is attached to a spring and slides without friction along a circular rod in a vertical plane. The spring has an undeformed length of 5 in. and a constant $k = 10$ lb/ft. Knowing that the collar is released from being held at $A$ determine the speed of the collar and the normal force between the collar and the rod as the collar passes through $B$.

**SOLUTION**

For the collar, 

$$m = \frac{W}{g} = \frac{1}{32.2} = 0.031056 \text{ lb} \cdot \text{s}^2/\text{ft}$$

For the spring, 

$$k = 10 \text{ lb/ft} \quad l_0 = 5 \text{ in.}$$

At $A$: 

$$\ell_A = 7 + 5 + 5 = 17 \text{ in.}$$

$$\ell_A - \ell_0 = 12 \text{ in.} = 1 \text{ ft}$$

At $B$: 

$$\ell_B = \sqrt{(7 + 5)^2 + 5^2} = 13 \text{ in.}$$

$$\ell_B - \ell_0 = 1.8 \text{ in.} = \frac{2}{3} \text{ ft}$$

Velocity of the collar at $B$.

Use the principle of conservation of energy. 

$$T_A + V_A = T_B + V_B$$

Where 

$$T_A = \frac{1}{2}mv_A^2 = 0$$

$$V_A = \frac{1}{2}k(\ell_A - \ell_0)^2 + W(0)$$

$$= \frac{1}{2}(10)(1)^2 + 0 = 5 \text{ ft} \cdot \text{lb}$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.031056)v_B^2 = 0.015528v_B^2$$

$$V_B = \frac{1}{2}k(\ell_B - \ell_0)^2 + Wh$$

$$= \frac{1}{2}(10)\left(\frac{2}{3}\right)^2 + (1)\left(-\frac{5}{12}\right)$$

$$= 1.80556 \text{ ft} \cdot \text{lb}$$

$$0 + 5 = 0.015528v_B^2 = 1.80556$$

$$v_B^2 = 205.72 \text{ ft}^2/\text{s}^2$$

$$v_B = 14.34 \text{ ft/s}$$

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Forces at B.

\[ F_s = k(\ell_B - \ell_0) = (10)\left(\frac{2}{3}\right) = 6.6667 \text{ lb.} \]

\[ \sin\alpha = \frac{5}{13} \]

\[ \rho = 5 \text{ in.} = \frac{5}{12} \text{ ft} \]

\[ ma_n = \frac{mv_B^2}{\rho} \]

\[ = \frac{(0.031056)(205.72)}{5/12} \]

\[ = 15.3332 \text{ lb} \]

\[ + \sum F_y = ma_n \quad F_s \sin\alpha - W + N = ma_n \]

\[ N = ma_n + W - F_s \sin\alpha \]

\[ = 15.3332 + 1 - (6.6667)\left(\frac{5}{13}\right) \]

\[ N = 13.769 \text{ lb} \]

\[ N = 13.77 \text{ lb} \]

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